THE IMPACT OF MALICIOUS AGENTS ON THE ENTERPRISE SOFTWARE INDUSTRY

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Appendix
Proofs

Proof of Proposition 1

The proof of this proposition follows from Lemmas 1 to 4 presented below.

**Lemma 1.** In equilibrium, (i) \( n_1 > 0 \) and (ii) \( n_2 = 0 \) \( \Rightarrow \) \( n_1 = 1 \)

**Proof.** (i) Assume that \( n_1 = 0 \). If consumers in the neighborhood of Firm 1 are not purchasing from Firm 2, then any price \( p_1 < v_1 \) will lead to positive sales and profits. Thus, assume that all consumers are purchasing from Firm 2. Firm 1 can make positive profits if there exists a \( p_1 > 0 \) such that \( u_1(0) > u_2(0) \), which is equivalent to:

\[
v_1 - p_1 > v_2 - p_2 - t - qL \equiv p_1 < v_1 - v_2 + p_2 + t + qL
\]

Since \( v_1 > v_2, t > 0 \), and, in equilibrium, we must have \( p_2 \geq 0 \), a \( p_1 > 0 \) satisfying the above condition must exist. (ii)

If consumers in the neighborhood of Firm 2 are not purchasing from Firm 1, then any price \( p_2 < v_2 \) will lead to positive sales and profits. \( \Box \)

**Lemma 2.** In equilibrium, when \( p_1 - p_2 \leq (v_1 - v_2) - t - qL \), the consumer adoption decision is: \( n_1 = 1, n_2 = 0 \).

**Proof.**

\[
\alpha_{10}^* = \frac{1}{t} (v_1 - p_1 - qLN_1^e)
\]

\[
\geq 1 + \frac{v_2 - p_2}{t} + \frac{qL}{t} (1 - n_1^e)
\]

Thus, by Equations (6) and (7), \( n_1 = 1, n_2 = 0 \) for any \( n_1^e, n_2^e \). \( \Box \)

**Lemma 3.** In equilibrium, when \( (v_1 - v_2) + p_2 - t - qL < p_1 \leq (v_1 + v_2) - p_2 - t - qL \), the consumer adoption decision satisfies: \( n_1 + n_2 = 1, n_2 > 0 \).
Proof.

\[
\alpha_{12}^* = \frac{1}{2} + \frac{1}{2t} \left[ (v_1 - v_2) - (p_1 - p_2) - qL(n_1^c - n_2^c) \right]
\]

\[
\geq 1 - \frac{1}{t}(v_2 - p_2) + \frac{qL}{2t} (1 - n_1^c + n_2^c)
\]

\[
\geq 1 - \frac{1}{t}(v_2 - p_2 - qLn_2^c)
\]

\[
= \alpha_{20}^*
\]

where the second inequality arises because \(n_1 + n_2 \leq 1\). Similar reasoning demonstrates that \(\alpha_{12}^* \leq \alpha_{10}^*\). By Equations (6) and (7), \(n_1 + n_2 = 1\). To demonstrate that both firms have positive market share, we must show that \(\alpha_{12}^* < 1\). Assume that \(\alpha_{12}^* \geq 1\). This implies that \(n_1 = 1, n_2 = 0\):

\[
\alpha_{12}^* = \frac{1}{2} + \frac{1}{2t} \left[ (v_1 - v_2) - (p_1 - p_2) - qL(n_1^c - n_2^c) \right]
\]

\[
< 1 + \frac{qL}{2t} (1 - n_1^c + n_2^c)
\]

\[
= 1
\]

which is a contradiction.

\[\Box\]

**Lemma 4.** In equilibrium, when \(p_1 + p_2 > (v_1 + v_2) - t - qL\), the consumer adoption decision satisfies: \(n_1 + n_2 < 1\) and \(n_2 > 0\).

**Proof.** Reversing the proof for the first step of Lemma 3 provides: \(\alpha_{10}^* < \alpha_{12}^* < \alpha_{20}^*\) which, by Equations (6) and (7), provides the first part of the lemma, that \(n_1 + n_2 < 1\). Further, \(n_2 = 1 - \alpha_{20}^*\). When \(n_2 = n_2^c\), we have \(n_2 = \frac{v_2 - p_2}{t+qL}\). That \(n_2 > 0\) follows from Firm 2’s profit maximization since any \(p_2 < v_2\) results in positive profit. \[\Box\]
Proof of Proposition 2

Prices follow from Lemma 5 below, and market shares follow from Equations (6) and (7).

Lemma 5. Firm $i$’s best response for any price of Firm $j$ is given by

$$p_i(p_j) = \begin{cases} 
  v_i - v_j + p_j - t - qL & \text{if } q < \frac{v_i - (v_j - p_j) - 3t}{3L} \\
  \frac{1}{2}(v_i - v_j + p_j + t + qL) & \text{if } \frac{v_i - (v_j - p_j) - 3t}{3L} \leq q \leq \frac{v_i + 3(v_j - p_j) - 3t}{3L} \\
  v_i + v_j - p_j - t - qL & \text{if } \frac{v_i + 3(v_j - p_j) - 3t}{3L} < q \leq \frac{v_i + 2(v_j - p_j) - 2t}{2L} \\
  \frac{1}{2}v_1 & \text{if } \frac{v_i + 2(v_j - p_j) - 2t}{2L} < q < v_i + \frac{3(v_j - p_j)}{2L} - t - qL \\
  1 & \text{if } p_j \leq v_j - v_i - t - qL \\
\end{cases}$$

Proof. From Proposition 1 and Equations (6) and (7), we know that:

$$n_1 = \begin{cases} 
  1 & \text{if } p_1 - p_2 \leq (v_1 - v_2) - t - qL \\
  \frac{1}{2} + \frac{(v_1 - p_1) - (v_2 - p_2)}{2(t + qL)} & \text{if } p_1 - p_2 > (v_1 - v_2) - t - qL \\
  \frac{v_1 - p_1}{t + qL} & \text{if } p_1 + p_2 \leq (v_1 + v_2) - t - qL \\
  \frac{v_1 - v_2 + p_1 + t + qL - 2v_1}{2(t + qL)} & \text{if } p_1 + p_2 > (v_1 + v_2) - t - qL \\
\end{cases}$$

The corresponding derivatives of Firm 1’s profit with respect to its price are:

$$\frac{\partial \pi_1(p_1)}{\partial p_1} = \begin{cases} 
  1 & \text{Region 1} \\
  \frac{v_1 - (v_2 - p_2) + t + qL - 2v_1}{2(t + qL)} & \text{Region 2} \\
  \frac{v_1 - 2p_1}{t + qL} & \text{Region 3} \\
\end{cases}$$

The regions are numbered for ease of discourse. Profit is increasing over Region 1. Inspection of the derivatives reveals four possibilities: (i) profit is decreasing in Regions 2 and 3, (ii) profit is single-peaked in the interior of Region 2 and decreasing in Region 3, (iii) profit is increasing in Region 2 and decreasing in Region 3, and (iv) profit is increasing in Region 2 and is single-peaked in Region 3. These correspond to the first four cases in the lemma. In the fifth case, when $p_j \leq v_j - v_i - t - qL$, Firm $i$ cannot obtain positive market share at any price. Best responses for Firm 2 are obtained analogously. 

\[\square\]
Proof of Theorems

The following lemma, defining conditions under which equilibrium profit is increasing in $q$, is used in the proofs of the theorems.

**Lemma 6.** For $j \in \{1, 2\}$,

(i) if $q \leq \frac{v_1 - v_2 - 3t}{3L}$, \( \frac{d\pi^*_j}{dq} \leq 0 \),

(ii) if $\frac{v_1 - v_2 - 3t}{3L} < q \leq \frac{v_1 + v_2 - 3t}{3L}$, \( \frac{d\pi^*_j}{dq} > 0 \),

(iii) if $\frac{v_1 + v_2 - 2t}{2L} < q$, \( \frac{d\pi^*_j}{dq} < 0 \).

(iv) if $\frac{v_1 + v_2 - 3t}{3L} < q \leq \frac{v_1 + v_2 - 2t}{2L}$, \( \frac{d\pi^*_j}{dq} < 0 \) and \( \frac{d\pi^*_j}{dq} < 0 \).

where $\pi^*_j$ and $\pi^*_j$ are the highest and lowest obtainable equilibrium profits for firm $j$.

**Proof.** Equilibrium profits, $\pi^*_j = n_jp_j$, are obtained from Proposition 2.

(i) Profits are given by $\pi^*_1 = v_1 - v_2 - t - qL$ and $\pi^*_2 = 0$, which are non-increasing in $q$.

(ii) Profits are given by:

$\pi^*_j = \frac{1}{2}(t + qL)\left(1 + \frac{(v_j - v_i)}{\pi(t+qL)}\right)^2$, $i \neq j$. Differentiating,

\[
\frac{d\pi^*_j}{dq} = \frac{1}{2}L\left[1 - \left(\frac{v_j - v_i}{3(t+qL)}\right)^2\right]
\]

which is positive whenever: $q > \frac{v_j - v_i - 3t}{3L}$

(iii) Profits are given by $\pi^*_j = \frac{v_j^2}{4(t+qL)}$ which is decreasing in $q$.

(iv) Profits are given by $\pi_j = \frac{(v_j - p_j)p_j}{t + qL}$. Differentiating with respect to $q$ yields:

\[
\frac{d\pi_j}{dq} = \left(\frac{v_j - 2p_j}{t + qL}\right)\frac{dp_j}{dq} - \frac{(v_j - p_j)p_j}{(t + qL)^2}
\]  \( (A-1) \)

By Equation (9c), the set of prices that can yield either the highest or lowest equilibrium payoffs for Firm 1 is $p_1 \in \{\frac{1}{2}v_1, \frac{2}{3}v_1, v_1 + \frac{1}{2}v_2 - t - qL, v_1 + \frac{1}{2}v_2 - t - qL\}$. In the first two cases, $\frac{dp_1}{dq} = 0$ and Equation (A-1) is negative. In the last two cases, Equation (A-1) becomes:

\[
\frac{d\pi_1(\{\pi^*_1, \pi^*_1\})}{dq} = -\frac{L}{(t + qL)^2}\left[(t + qL)^2 - (sv_2)^2 - sv_1v_2\right]
\]
where \( s \in \{ \frac{1}{3}, \frac{1}{2} \} \). To complete the proof, we show that the part in brackets is positive.

\[
(t + qL)^2 - (sv_2)^2 - sv_1v_2 \geq (t + qL)^2 - \frac{v_2^2}{9} - \frac{v_1v_2}{3}
\]

\[
> \left( \frac{v_1 + v_2}{3} \right)^2 - \frac{v_2^2}{9} - \frac{v_1v_2}{3}
\]

\[
= \frac{v_1}{9} (v_1 - v_2) > 0
\]

**Proof of Theorem 1.** (i) The condition \( t \leq \frac{1}{3}(v_1 - v_2) - L \) implies that \( q \leq \frac{v_1 - v_2 - 3t}{3L} \) for all \( q \in [0, 1] \). By Lemma 6, we have that \( \frac{d\pi_j^*}{dq} \leq 0 \).

(ii) The condition is equivalent to \( \frac{v_1 + v_2 - 2t}{2L} < 0 \) which implies that \( q > \frac{v_1 + v_2 - 2t}{2L} \). By Lemma 6, we have that \( \frac{d\pi_j^*}{dq} < 0 \).

(iii) The condition \( t > \frac{1}{3}(v_1 + v_2) \) implies that \( q > \frac{v_1 + v_2 - 3t}{3L} \) for all \( q \in [0, 1] \). By Lemma 6, we have that \( \pi_j^*, \pi_j^* < 0 \).

**Proof of Theorem 2.** By Proposition 2:

\( n_2 = 0 \) if \( q \leq \frac{v_1 - v_2 - 3t}{3L} \) and \( n_2 > 0 \) if \( q > \frac{v_1 - v_2 - 3t}{3L} \)

For part (i) of the theorem, to have \( n_2 = 0 \) when \( q = 0 \), we need \( 0 \leq \frac{v_1 - v_2 - 3t}{3L} \). For part (ii), we require that \( 1 > \frac{v_1 - v_2 - 3t}{3L} \). These conditions are equivalent to:

\[
\frac{1}{3}(v_1 - v_2) - L < t \leq \frac{1}{3}(v_1 - v_2)
\]

**Proof of Theorem 3.** Define \( q \equiv \max[0, \frac{v_1 - v_2 - 3t}{3L}] \) and \( \overline{q} \equiv \min[1, \frac{v_1 + v_2 - 3t}{3L}] \). Clearly, \( q \geq 0 \) and \( \overline{q} \leq 1 \) and, by Lemma 6, profit is increasing whenever \( q \in (q, \overline{q}) \). To show that \( q < \overline{q} \) we require:

\[
1 > \frac{v_1 - v_2 - 3t}{3L} \quad \text{and} \quad 0 < \frac{v_1 + v_2 - 3t}{3L}
\]

\[
\equiv \quad t > \frac{1}{3}(v_1 - v_2) - L \quad \text{and} \quad t < \frac{1}{3}(v_1 + v_2)
\]

which correspond to the conditions of part (i) of the theorem. The conditions in part (ii) imply

\[
t \leq \frac{1}{3}(v_1 + v_2) - L \quad \Rightarrow \quad \frac{v_1 + v_2 - 3t}{3L} \geq 1
\]

\[
t > \frac{1}{3}(v_1 - v_2) \quad \Rightarrow \quad \frac{v_1 - v_2 - 3t}{3L} < 0
\]

Therefore, \( \frac{v_1 - v_2 - 3t}{3L} < q \leq \frac{v_1 + v_2 - 3t}{3L} \) which, by Lemma 6, implies profit is increasing for all \( q \). \( \square \)
We next consider the generality of the above results, by specifying a quadratic attack probability function which includes linearity as a special case.

**Corollary 3 (quadratic attack probability).** Define

\[ q(n_j) \equiv q\beta n_j + q(1-\beta)n_j^2 \]  

(A-2)

Where \( \beta \in [0, 1] \). If

(i) \( t > \frac{1}{3}(v_1 - v_2) \), and

(ii) \( v_1 \) and \( v_2 \) are sufficiently large so that every consumer derives strictly positive utility in equilibrium when \( q = 0 \),

Then, both firms obtain maximal profit at some \( q > 0 \).

**Proof.** The consumer indifferent between Firm 1 and Firm 2 is found by solving:

\[ u_1(\alpha^{*}_{12}) = u_2(\alpha^{*}_{12}) \]

\[ \alpha^{*}_{12} = \frac{1}{2} + \frac{(v_1-v_2)-(p_1-p_2)}{2t} - \frac{qL}{2t} \left[ \beta(n_1^c - n_2^c) + (1-\beta) \left( (n_1^c)^2 - (n_2^c)^2 \right) \right] \]  

(A-3)

Since all consumers derive strictly positive utility when \( q = 0 \), by assumption, we must have \( n_1^c + n_2^c = 1 \) when \( q \) is sufficiently small. Equation (A-3) becomes:

\[ \alpha^{*}_{12} = \frac{1}{2} + \frac{(v_1-v_2)-(p_1-p_2)}{2t} - qL \left( 2n_1^c - 1 \right) \]  

(A-4)

In equilibrium, it must be the case that \( \alpha^{*}_{12} = n_1 = n_2^c \). Substituting into (A-4) yields

\[ n_1 = \frac{1}{2} + \frac{(v_1-v_2)-(p_1-p_2)}{2(t+qL)} \]  

(A-5)

For the above to have an interior solution \((0 < n_1 < 1)\), we must have:

\[ (v_1 - v_2) - (t + qL) < p_1 - p_2 < (v_1 - v_2) + (t + qL) \]  

(A-6)

We will confirm these conditions shortly. First, equilibrium prices are obtained by differentiating \( \pi_j = p_jn_j \) for each firm and solving the simultaneous equations. This yields:

\[ p_j = \frac{1}{2}(v_j - v_i) + t + qL, \quad i, j \in \{1, 2\}, i \neq j \]  

(A-7)

The conditions in (A-6) are satisfied whenever \( t + qL > \frac{1}{3}(v_1 - v_2) \) which is true by assumption.
(condition i). Combining (A-5) and (A-7) yields profits of:

$$\pi_j = \frac{[t + qL + \frac{1}{3}(v_1 - v_2)]^2}{2(t + qL)}$$

which is increasing in $q$ whenever $t > \frac{1}{3}(v_1 - v_2)$.

**Proof of Theorem 4.** Condition (i) guarantees that the profit function is initially increasing in $q$. In particular, it implies that

$$\frac{v_1 - v_2 - 3t}{3L} < 0 \leq \frac{v_1 + v_2 - 3t}{3L}$$

which, by Lemma 6, implies that $\frac{d\pi^*_j}{dq} \bigg|_{q=0} > 0$.

If the profit function is initially nonincreasing, then there are two possibilities by Lemma 6. As $q$ increases, either profit is initially nonincreasing, then increasing; or it is nonincreasing, then increasing, then decreasing:

**(ii-a) nonincreasing-increasing:** By Lemma 6, for profits to be nonincreasing when $q = 0$ and increasing when $q = 1$, the following conditions are required:

$$0 \leq \frac{v_1 - v_2 - 3t}{3L} \Rightarrow t \leq \frac{1}{3}(v_1 - v_2) \quad (A-8)$$

$$1 > \frac{v_1 - v_2 - 3t}{3L} \Rightarrow t > \frac{1}{3}(v_1 - v_2) - L \quad (A-9)$$

$$1 \leq \frac{v_1 + v_2 - 3t}{3L} \Rightarrow t \leq \frac{1}{3}(v_1 + v_2) - L \quad (A-10)$$

Firm 2’s profit is 0 at $q = 0$, thus Firm 2’s profit is maximized at $q = 1$. For Firm 1, maximum profit occurs either at $q = 0$ or $q = 1$ and, by Proposition 2, these are given by:

$$\pi^*_1|_{q=0} = v_1 - v_2 - t$$

$$\pi^*_1|_{q=1} = \frac{1}{2(t+L)} [\frac{1}{3}(v_1 - v_2) + t + L]^2 \quad (A-11)$$

Profit at $q = 1$ is strictly greater than profit at $q = 0$ when

$$L > 2 \left( \frac{1}{3}v_1 - \frac{1}{3}v_2 - t \right) + \sqrt{\left( \frac{1}{3}v_1 - \frac{1}{3}v_2 - t \right) (v_1 - v_2 - t)} \quad (A-12)$$

Condition (A-9) is redundant as it is implied by (A-13). However, for both (A-13) and (A-10) to be satisfied, it must also be the case that

$$t > \frac{v_1^2 - 3v_2^2}{3(v_1 + v_2)} \quad (A-14)$$
Combining these conditions:
\[
\frac{1}{3}(v_1 - v_2) \geq t > \frac{v_1^2 - 3v_2^2}{3(v_1 + v_2)} \tag{A-15}
\]
\[
\frac{1}{3}(v_1 + v_2) - t \geq L > 2 \left( \frac{1}{3}v_1 - \frac{1}{3}v_2 - t \right) (v_1 - v_2 - t)
\]

(ii-b) nonincreasing-increasing-decreasing: By Lemma 6, we require:
\[
0 \leq \frac{v_1 - v_2 - 3t}{3L} \Rightarrow t \leq \frac{1}{3}(v_1 - v_2) \tag{A-16}
\]
\[
1 > \frac{v_1 + v_2 - 3t}{3L} \Rightarrow t > \frac{1}{3}(v_1 + v_2) - L \tag{A-17}
\]

Maximum profit can occur either at \( q = 0 \) or at \( q = \frac{v_1 + v_2 - 3t}{3L} \) which is the point above which profit is again decreasing in \( q \). Firm 1’s profit is given by:
\[
\pi_1^*|_{q=\frac{v_1 + v_2 - 3t}{3L}} = \frac{2v_1^2}{3(v_1 + v_2)} \tag{A-18}
\]

This profit exceeds the profit at \( q = 0 \) given by (A-11) if \( t > \frac{v_1^2 - 3v_2^2}{3(v_1 + v_2)} \) which is precisely the condition in (A-14). Combining these conditions, we have:
\[
\frac{1}{3}(v_1 - v_2) \geq t > \frac{v_1^2 - 3v_2^2}{3(v_1 + v_2)} \tag{A-19}
\]
\[
L > \frac{1}{3}(v_1 + v_2) - t
\]

Taking the union of parameter ranges in (A-15) and (A-19) yields condition (ii) in the theorem.

Proof of Theorem 5. We solve for the subgame perfect equilibrium. The consumer indifferent between Firm 1 and Firm 2 is given by
\[
\alpha_{12}^* = \frac{1}{2} + \frac{1}{2t} \left[ (v_1 - v_2) - (p_1 - p_2) - (q_1n_1^e - q_2n_2^e)L \right] \tag{A-20}
\]

Following steps similar to Propositions 1 and 2, under the conditions in the theorem, we have \( n_1 > 0, n_2 > 0, n_1 + n_2 = 1 \) for all \( q_1 \) and \( q_2 \). Since, in equilibrium, \( n_i^e = n_i \), we have
\[
n_1 = \frac{(v_1 - v_2) - (p_1 - p_2) + t + q_2L}{2t + (q_1 + q_2)L} \tag{A-21}
\]

For given \( q_1 \) and \( q_2 \), firms maximize \( \pi_i(p_i) = p_in_i \) which yields the first order conditions:
\[
p_i = \frac{1}{2} (v_i - v_j + p_j + t + q_jL) \quad i, j \in \{1, 2\}, i \neq j \tag{A-22}
\]
From these, the equilibrium prices and market shares are given by:

\[
p_1 = \frac{1}{3} (v_1 - v_2) + t + \frac{1}{3} (q_1 + 2q_2)L, \quad p_2 = \frac{1}{3} (v_2 - v_1) + t + \frac{1}{3} (q_2 + 2q_1)L \quad (A-23)
\]

\[
n_1 = \frac{(v_1 - v_2) + 3t + (q_1 + 2q_2)L}{6t + 3(q_1 + q_2)L}, \quad n_2 = 1 - n_1 \quad (A-24)
\]

In the first stage, firms select \( q_i \) to maximize \( p_i n_i - c_i(q_i) \). For \( i, j \in \{1, 2\}, i \neq j \), profit as a function of \( q_i, q_j \) is given by:

\[
\pi_i(q_i, q_j) = \left( (v_i - v_j) + 3t + (q_i + 2q_j)L \right) - c_i(q_i) \quad (A-25)
\]

Taking the derivative with respect to \( q_i \) yields:

\[
\frac{d\pi_i(q_i, q_j)}{dq_i} = \left[ v_j - v_i + t + q_iL \left( \frac{L(v_i - v_j) + 3t + (q_i + 2q_j)L}{9(2t + (q_i + q_j)L)^2} \right) \right] - c_i'(q_i) \quad (A-26)
\]

The fraction term is strictly positive since \( t > \frac{1}{3} (v_1 - v_2) \). Also, \( c_i'(q_i) \leq 0 \).

(i) For an equilibrium to satisfy \( q_1 = q_2 = 0 \), it must be the case that the derivative of each firm’s profit function at \( q_1 = q_2 = 0 \) must be non-positive. Consider firm 2. The expression in the brackets becomes \( [v_1 - v_2 + t] > 0 \). Therefore, the derivative is positive.

(ii) For \( q_i = 1 \) to be a dominant strategy, the derivative of profit must be increasing for all \( q_i, q_j \). This requires that the expression in the square brackets be positive, which is true whenever \( t > v_1 - v_2 \). □

**Proof of Theorem 6.** The consumer indifferent between Firms 1 and 2 \( (u_1 = u_2) \) is given by

\[
\alpha_{12}^* = \frac{1}{\Delta} \left[ (p_1 - p_2) + qL(n_1^* - n_2^*) \right] \quad (A-27)
\]

By assumption, \( n_1 + n_2 = 1 \) and therefore \( n_1 = 1 - \alpha_{12}^* \). In equilibrium, \( n_j^* = n_j \), implying

\[
n_1 = \frac{\Delta - (p_1 - p_2) + qL}{\Delta + 2qL} \quad (A-28)
\]

Maximizing each firm’s profit, \( n_j p_j \), with respect to \( p_j \) and substituting yields the expressions in (12) and (13). As both \( p_2 \) and \( n_2 \) are increasing in \( q \), the result holds for Firm 2. For Firm 1, profits are given by \( p_1 n_1 = \frac{1}{9} \left( 2\Delta + 3qL \right)^2 \). Differentiating with respect to \( q \) yields the result. □