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THE IMPACT OF MALICIOUS AGENTS ON THE ENTERPRISE SOFTWARE INDUSTRY

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Appendix

Proofs

Proof of Proposition 1

The proof of this proposition follows from Lemmas 1 to 4 presented below.

Lemma 1. *In equilibrium, (i)* $n_1 > 0$ *and (ii)* $n_2 = 0 \Rightarrow n_1 = 1$

Proof. (i) Assume that $n_1 = 0$. If consumers in the neighborhood of Firm 1 are not purchasing from Firm 2, then any price $p_1 < v_1$ will lead to positive sales and profits. Thus, assume that all consumers are purchasing from Firm 2. Firm 1 can make positive profits if there exists a $p_1 > 0$ such that $u_1(0) > u_2(0)$, which is equivalent to:

$$v_1 - p_1 > v_2 - p_2 - t - qL \equiv p_1 < v_1 - v_2 + p_2 + t + qL$$

Since $v_1 > v_2$, $t > 0$, and, in equilibrium, we must have $p_2 \geq 0$, a $p_1 > 0$ satisfying the above condition must exist. (ii)

If consumers in the neighborhood of Firm 2 are not purchasing from Firm 1, then any price $p_2 < v_2$ will lead to positive sales and profits. □

Lemma 2. *In equilibrium, when* $p_1 - p_2 \leq (v_1 - v_2) - t - qL$*, the consumer adoption decision is:* $n_1 = 1$, $n_2 = 0$.

Proof.

$$\begin{aligned}
\alpha_{12}^* &= \frac{1}{2} + \frac{1}{2t}\left[(v_1 - v_2) - (p_1 - p_2) - qL(n_1^e - n_2^e)\right] \\
&\geq 1 + \frac{qL}{2t}(1 - n_1^e + n_2^e) \\
&\geq 1 \\
\alpha_{10}^* &= \frac{1}{t}\left(v_1 - p_1 - qLn_1^e\right) \\
&\geq 1 + \frac{v_2 - p_2}{t} + \frac{qL}{t}(1 - n_1^e) \\
&\geq 1
\end{aligned}$$

Thus, by Equations (6) and (7), $n_1 = 1, n_2 = 0$ for any n_1^e, n_2^e. □

Lemma 3. *In equilibrium, when* $(v_1 - v_2) + p_2 - t - qL < p_1 \leq (v_1 + v_2) - p_2 - t - qL$*, the consumer adoption decision satisfies:* $n_1 + n_2 = 1, n_2 > 0$.

Proof.

$$\begin{aligned}
\alpha_{12}^* &= \frac{1}{2} + \frac{1}{2t}\left[(v_1 - v_2) - (p_1 - p_2) - qL(n_1^e - n_2^e)\right] \\
&\geq 1 - \frac{1}{t}(v_2 - p_2) + \frac{qL}{2t}(1 - n_1^e + n_2^e) \\
&\geq 1 - \frac{1}{t}(v_2 - p_2 - qLn_2^e) \\
&= \alpha_{20}^*
\end{aligned}$$

where the second inequality arises because $n_1 + n_2 \leq 1$. Similar reasoning demonstrates that $\alpha_{12}^* \leq \alpha_{10}^*$. By Equations (6) and (7), $n_1 + n_2 = 1$. To demonstrate that both firms have positive market share, we must show that $\alpha_{12}^* < 1$. Assume that $\alpha_{12}^* \geq 1$. This implies that $n_1 = 1, n_2 = 0$:

$$\begin{aligned}
\alpha_{12}^* &= \frac{1}{2} + \frac{1}{2t}\left[(v_1 - v_2) - (p_1 - p_2) - qL(n_1^e - n_2^e)\right] \\
&< 1 + \frac{qL}{2t}\left(1 - n_1^e + n_2^e\right) \\
&= 1
\end{aligned}$$

which is a contradiction. □

Lemma 4. *In equilibrium, when $p_1 + p_2 > (v_1 + v_2) - t - qL$, the consumer adoption decision satisfies: $n_1 + n_2 < 1$ and $n_2 > 0$.*

Proof. Reversing the proof for the first step of Lemma 3 provides: $\alpha_{10}^* < \alpha_{12}^* < \alpha_{20}^*$ which, by Equations (6) and (7), provides the first part of the lemma, that $n_1 + n_2 < 1$. Further, $n_2 = 1 - \alpha_{20}^*$. When $n_2 = n_2^e$, we have $n_2 = \frac{v_2 - p_2}{t + qL}$. That $n_2 > 0$ follows from Firm 2's profit maximization since any $p_2 < v_2$ results in positive profit. □

Proof of Proposition 2

Prices follow from Lemma 5 below, and market shares follow from Equations (6) and (7).

Lemma 5. *Firm i's best response for any price of Firm j is given by*

$$p_i(p_j) = \begin{cases} v_i - v_j + p_j - t - qL & \text{if} \quad q < \frac{v_i - (v_j - p_j) - 3t}{3L} \\ \frac{1}{2}(v_i - v_j + p_j + t + qL) & \text{if} \quad \frac{v_i - (v_j - p_j) - 3t}{3L} \le q \le \frac{v_i + 3(v_j - p_j) - 3t}{3L} \\ v_i + v_j - p_j - t - qL & \text{if} \quad \frac{v_i + 3(v_j - p_j) - 3t}{3L} < q \le \frac{v_i + 2(v_j - p_j) - 2t}{2L} \\ \frac{1}{2}v_1 & \text{if} \quad \frac{v_i + 2(v_j - p_j) - 2t}{2L} < q \end{cases}$$

$$p_i(p_j) \in [0, \infty) \quad \text{if} \quad p_j \le v_j - v_i - t - qL$$

Proof. From Proposition 1 and Equations (6) and (7), we know that:

$$n_1 = \begin{cases} 1 & \text{if} \quad p_1 - p_2 \le (v_1 - v_2) - t - qL \\ \frac{1}{2} + \frac{(v_1 - p_1) - (v_2 - p_2)}{2(t + qL)} & \text{if} \quad p_1 - p_2 > (v_1 - v_2) - t - qL \\ & \phantom{\text{if}} \quad p_1 + p_2 \le (v_1 + v_2) - t - qL \\ \frac{v_1 - p_1}{t + qL} & \text{if} \quad p_1 + p_2 > (v_1 + v_2) - t - qL \end{cases}$$

The corresponding derivatives of Firm 1's profit with respect to its price are:

$$\frac{\partial \pi_1(p_1)}{\partial p_1} = \begin{cases} 1 & \text{Region 1} \\ & p_1 \le v_1 - (v_2 - p_2) - t - qL \\ \frac{v_1 - (v_2 - p_2) + t + qL - 2p_1}{2(t + qL)} & \text{Region 2} \\ & v_1 - (v_2 - p_2) - t - qL < p_1 \le v_1 + (v_2 - p_2) - t - qL \\ \frac{v_1 - 2p_1}{t + qL} & \text{Region 3} \\ & v_1 + (v_2 - p_2) - t - qL < p_1 \le v_1 \end{cases}$$

The regions are numbered for ease of discourse. Profit is increasing over Region 1. Inspection of the derivatives reveals four possibilities: (i) profit is decreasing in Regions 2 and 3, (ii) profit is single-peaked in the interior of Region 2 and decreasing in Region 3, (iii) profit is increasing in Region 2 and decreasing in Region 3, and (iv) profit is increasing in Region 2 and is single-peaked in Region 3. These correspond to the first four cases in the lemma. In the fifth case, when $p_j \le v_j - v_i - t - qL$, Firm i cannot obtain positive market share at any price. Best responses for Firm 2 are obtained analogously. □

Proof of Theorems

The following lemma, defining conditions under which equilibrium profit is increasing in q, is used in the proofs of the theorems.

Lemma 6. *For $j \in \{1,2\}$,*

$$
\begin{array}{llll}
(i) & \text{if} & q \leq \frac{v_1-v_2-3t}{3L}, & \frac{d\pi_j^*}{dq} \leq 0, \\
(ii) & \text{if} & \frac{v_1-v_2-3t}{3L} < q \leq \frac{v_1+v_2-3t}{3L}, & \frac{d\pi_j^*}{dq} > 0, \\
(iii) & \text{if} & \frac{v_1+v_2-2t}{2L} < q, & \frac{d\pi_j^*}{dq} < 0. \\
(iv) & \text{if} & \frac{v_1+v_2-3t}{3L} < q \leq \frac{v_1+v_2-2t}{2L}, & \frac{d\overline{\pi}_j^*}{dq} < 0 \text{ and } \frac{d\underline{\pi}_j^*}{dq} < 0.
\end{array}
$$

where $\overline{\pi}_j^*$ and $\underline{\pi}_j^*$ are the highest and lowest obtainable equilibrium profits for firm j.

Proof. Equilibrium profits, $\pi_j^* = n_j p_j$, are obtained from Proposition 2.
(i) Profits are given by $\pi_1^* = v_1 - v_2 - t - qL$ and $\pi_2^* = 0$, which are nonincreasing in q.
(ii) Profits are given by: $\pi_j^* = \frac{1}{2}(t+qL)\left(1 + \frac{v_j - v_i}{3(t+qL)}\right)^2$, $i \neq j$. Differentiating,

$$\frac{d\pi_j^*}{dq} = \frac{1}{2}L\left[1 - \left(\frac{v_j - v_i}{3(t+qL)}\right)^2\right]$$

which is positive whenever: $q > \frac{v_j - v_i - 3t}{3L}$
(iii) Profits are given by $\pi_j^* = \frac{v_j^2}{4(t+ql)}$ which is decreasing in q.
(iv) Profits are given by $\pi_j = \frac{(v_j - p_j)p_j}{t+qL}$. Differentiating with respect to q yields:

$$\frac{d\pi_j}{dq} = \left(\frac{v_j - 2p_j}{t+qL}\right)\frac{dp_j}{dq} - \frac{(v_j-p_j)Lp_j}{(t+qL)^2} \qquad \text{(A-1)}$$

By Equation (9c), the set of prices that can yield either the highest or lowest equilibrium payoffs for Firm 1 is $p_1 \in \{\frac{1}{2}v_1, \frac{2}{3}v_1, v_1 + \frac{1}{3}v_2 - t - qL, v_1 + \frac{1}{2}v_2 - t - qL\}$. In the first two cases, $\frac{dp_1}{dq} = 0$ and Equation (A-1) is negative. In the last two cases, Equation (A-1) becomes:

$$\frac{d\pi_1(\in \{\overline{\pi}_1^*, \underline{\pi}_1^*\})}{dq} = -\frac{L}{(t+qL)^2}\left[(t+qL)^2 - (sv_2)^2 - sv_1v_2\right]$$

where $s \in \{\frac{1}{3}, \frac{1}{2}\}$. To complete the proof, we show that the part in brackets is positive.

$$\begin{aligned}(t+qL)^2 - (sv_2)^2 - sv_1v_2 &\geq (t+qL)^2 - \frac{v_2^2}{9} - \frac{v_1v_2}{3} \\ &> \left(\frac{v_1+v_2}{3}\right)^2 - \frac{v_2^2}{9} - \frac{v_1v_2}{3} \\ &= \frac{v_1}{9}(v_1 - v_2) > 0\end{aligned}$$

□

Proof of Theorem 1. (i) The condition $t \leq \frac{1}{3}(v_1 - v_2) - L$ implies that $q \leq \frac{v_1-v_2-3t}{3L}$ for all $q \in [0,1]$. By Lemma 6, we have that $\frac{d\pi_j^*}{dq} \leq 0$.
(ii) The condition is equivalent to $\frac{v_1+v_2-2t}{2L} < 0$ which implies that $q > \frac{v_1+v_2-2t}{2L}$. By Lemma 6, we have that $\frac{d\pi_j^*}{dq} < 0$.
(iii) The condition $t > \frac{1}{3}(v_1 + v_2)$ implies that $q > \frac{v_1+v_2-3t}{3L}$ for all $q \in [0,1]$. By Lemma 6, we have that $\overline{\pi}_j^*, \underline{\pi}_j^* < 0$. □

Proof of Theorem 2. By Proposition 2:

$$n_2 = 0 \text{ if } q \leq \tfrac{v_1-v_2-3t}{3L} \qquad \text{and} \qquad n_2 > 0 \text{ if } q > \tfrac{v_1-v_2-3t}{3L}$$

For part *(i)* of the theorem, to have $n_2 = 0$ when $q = 0$, we need $0 \leq \frac{v_1-v_2-3t}{3L}$. For part *(ii)*, we require that $1 > \frac{v_1-v_2-3t}{3L}$. These conditions are equivalent to:

$$\tfrac{1}{3}(v_1 - v_2) - L < t \leq \tfrac{1}{3}(v_1 - v_2)$$

□

Proof of Theorem 3. Define $\underline{q} \equiv \max[0, \frac{v_1-v_2-3t}{3L}]$ and $\overline{q} \equiv \min[1, \frac{v_1+v_2-3t}{3L}]$. Clearly, $\underline{q} \geq 0$ and $\overline{q} \leq 1$ and, by Lemma 6, profit is increasing whenever $q \in (\underline{q}, \overline{q})$. To show that $\underline{q} < \overline{q}$ we require:

$$\begin{array}{cccc} & 1 > \frac{v_1-v_2-3t}{3L} & \text{and} & 0 < \frac{v_1+v_2-3t}{3L} \\ \equiv & t > \frac{1}{3}(v_1 - v_2) - L & \text{and} & t < \frac{1}{3}(v_1 + v_2) \end{array}$$

which correspond to the conditions of part *(i)* of the theorem. The conditions in part *(ii)* imply

$$\begin{array}{ccc} t \leq \frac{1}{3}(v_1 + v_2) - L & \Rightarrow & \frac{v_1+v_2-3t}{3L} \geq 1 \\ t > \frac{1}{3}(v_1 - v_2) & \Rightarrow & \frac{v_1-v_2-3t}{3L} < 0 \end{array}$$

Therefore, $\frac{v_1-v_2-3t}{3L} < q \leq \frac{v_1+v_2-3t}{3L}$ which, by Lemma 6, implies profit is increasing for all q. □

We next consider the generality of the above results, by specifying a quadratic attack probability function which includes linearity as a special case.

Corollary 3 (quadratic attack probability). *Define*

$$q(n_j) \equiv q\beta n_j + q(1-\beta)n_j^2 \tag{A-2}$$

Where $\beta \in [0,1]$*. If*

(i) $t > \frac{1}{3}(v_1 - v_2)$*, and*

(ii) v_1 *and* v_2 *are sufficiently large so that every consumer derives strictly positive utility in equilibrium when* $q = 0$*,*

Then, both firms obtain maximal profit at some $q > 0$*.*

Proof. The consumer indifferent between Firm 1 and Firm 2 is found by solving:

$$\begin{aligned} & u_1(\alpha_{12}^*) = u_2(\alpha_{12}^*) \\ \equiv \quad & \alpha_{12}^* = \tfrac{1}{2} + \tfrac{(v_1-v_2)-(p_1-p_2)}{2t} - \tfrac{qL}{2t}\left[\beta(n_1^e - n_2^e) + (1-\beta)\left((n_1^e)^2 - (n_2^e)^2\right)\right] \end{aligned} \tag{A-3}$$

Since all consumers derive strictly positive utility when $q = 0$, by assumption, we must have $n_1^e + n_2^e = 1$ when q is sufficiently small. Equation (A-3) becomes:

$$\alpha_{12}^* = \tfrac{1}{2} + \tfrac{(v_1-v_2)-(p_1-p_2)}{2t} - \tfrac{qL}{2t}(2n_1^e - 1) \tag{A-4}$$

In equilibrium, it must be the case that $\alpha_{12}^* = n_1 = n_1^e$. Substituting into (A-4) yields

$$n_1 = \tfrac{1}{2} + \tfrac{(v_1-v_2)-(p_1-p_2)}{2(t+qL)} \tag{A-5}$$

For the above to have an interior solution ($0 < n_1 < 1$), we must have:

$$(v_1 - v_2) - (t + qL) < p_1 - p_2 < (v_1 - v_2) + (t + qL) \tag{A-6}$$

We will confirm these conditions shortly. First, equilibrium prices are obtained by differentiating $\pi_j = p_j n_j$ for each firm and solving the simultaneous equations. This yields:

$$p_j = \tfrac{1}{3}(v_j - v_i) + t + qL, \quad i, j \in \{1, 2\}, i \neq j \tag{A-7}$$

The conditions in (A-6) are satisfied whenever $t + qL > \frac{1}{3}(v_1 - v_2)$ which is true by assumption (condition *i*). Combining (A-5) and (A-7) yields profits of:

$$\pi_j = \frac{\left[t + qL + \frac{1}{3}(v_1 - v_2)\right]^2}{2(t + qL)}$$

which is increasing in q whenever $t > \frac{1}{3}(v_1 - v_2)$. □

Proof of Theorem 4. Condition *(i)* guarantees that the profit function is initially increasing in q. In particular, it implies that

$$\tfrac{v_1 - v_2 - 3t}{3L} < 0 \leq \tfrac{v_1 + v_2 - 3t}{3L}$$

which, by Lemma 6, implies that $\left.\frac{d\pi_j^*}{dq}\right|_{q=0} > 0$.

If the profit function is initially nonincreasing, then there are two possibilities by Lemma 6. As q increases, either profit is initially nonincreasing, then increasing; or it is nonincreasing, then increasing, then decreasing:

(ii-a) nonincreasing-increasing: By Lemma 6, for profits to be nonincreasing when $q = 0$ and increasing when $q = 1$, the following conditions are required:

$$0 \leq \tfrac{v_1 - v_2 - 3t}{3L} \quad \Rightarrow \quad t \leq \tfrac{1}{3}(v_1 - v_2) \tag{A-8}$$

$$1 > \tfrac{v_1 - v_2 - 3t}{3L} \quad \Rightarrow \quad t > \tfrac{1}{3}(v_1 - v_2) - L \tag{A-9}$$

$$1 \leq \tfrac{v_1 + v_2 - 3t}{3L} \quad \Rightarrow \quad t \leq \tfrac{1}{3}(v_1 + v_2) - L \tag{A-10}$$

Firm 2's profit is 0 at $q = 0$, thus Firm 2's profit is maximized at $q = 1$. For Firm 1, maximum profit occurs either at $q = 0$ or $q = 1$ and, by Proposition 2, these are given by:

$$\pi_1^*|_{q=0} = v_1 - v_2 - t \tag{A-11}$$

$$\pi_1^*|_{q=1} = \tfrac{1}{2(t+L)} \left[\tfrac{1}{3}(v_1 - v_2) + t + L\right]^2 \tag{A-12}$$

Profit at $q = 1$ is strictly greater than profit at $q = 0$ when

$$L > 2\left(\tfrac{1}{3}v_1 - \tfrac{1}{3}v_2 - t\right) + \sqrt{\left(\tfrac{1}{3}v_1 - \tfrac{1}{3}v_2 - t\right)(v_1 - v_2 - t)} \tag{A-13}$$

Condition (A-9) is redundant as it is implied by (A-13). However, for both (A-13) and (A-10) to be satisfied, it must also be the case that

$$t > \tfrac{v_1^2 - 3v_2^2}{3(v_1 + v_2)} \tag{A-14}$$

Combining these conditions:

$$\begin{aligned} \tfrac{1}{3}(v_1 - v_2) \geq t > \tfrac{v_1^2 - 3v_2^2}{3(v_1+v_2)} \\ \tfrac{1}{3}(v_1 + v_2) - t \geq L > 2\left(\tfrac{1}{3}v_1 - \tfrac{1}{3}v_2 - t\right) + \sqrt{\left(\tfrac{1}{3}v_1 - \tfrac{1}{3}v_2 - t\right)(v_1 - v_2 - t)} \end{aligned} \quad \text{(A-15)}$$

(ii-b) nonincreasing-increasing-decreasing: By Lemma 6, we require:

$$0 \leq \tfrac{v_1 - v_2 - 3t}{3L} \quad \Rightarrow \quad t \leq \tfrac{1}{3}(v_1 - v_2) \quad \text{(A-16)}$$

$$1 > \tfrac{v_1 + v_2 - 3t}{3L} \quad \Rightarrow \quad t > \tfrac{1}{3}(v_1 + v_2) - L \quad \text{(A-17)}$$

Maximum profit can occur either at $q = 0$ or at $q = \frac{v_1+v_2-3t}{3L}$ which is the point above which profit is again decreasing in q. Firm 1's profit is given by:

$$\pi_1^*\big|_{q=\frac{v_1+v_2-3t}{3L}} = \tfrac{2v_1^2}{3(v_1+v_2)} \quad \text{(A-18)}$$

This profit exceeds the profit at $q = 0$ given by (A-11) if $t > \frac{v_1^2 - 3v_2^2}{3(v_1+v_2)}$ which is precisely the condition in (A-14). Combining these conditions, we have:

$$\begin{aligned} \tfrac{1}{3}(v_1 - v_2) \geq t > \tfrac{v_1^2 - 3v_2^2}{3(v_1+v_2)} \\ L > \tfrac{1}{3}(v_1 + v_2) - t \end{aligned} \quad \text{(A-19)}$$

Taking the union of parameter ranges in (A-15) and (A-19) yields condition *(ii)* in the theorem.

□

Proof of Theorem 5. We solve for the subgame perfect equilibrium. The consumer indifferent between Firm 1 and Firm 2 is given by

$$\alpha_{12}^* = \frac{1}{2} + \frac{1}{2t}\left[(v_1 - v_2) - (p_1 - p_2) - (q_1 n_1^e - q_2 n_2^e)L\right] \quad \text{(A-20)}$$

Following steps similar to Propositions 1 and 2, under the conditions in the theorem, we have $n_1 > 0, n_2 > 0, n_1 + n_2 = 1$ for all q_1 and q_2. Since, in equilibrium, $n_i^e = n_i$, we have

$$n_1 = \frac{(v_1 - v_2) - (p_1 - p_2) + t + q_2 L}{2t + (q_1 + q_2)L} \quad \text{(A-21)}$$

For given q_1 and q_2, firms maximize $\pi_i(p_i) = p_i n_i$ which yields the first order conditions:

$$p_i = \frac{1}{2}\left(v_i - v_j + p_j + t + q_j L\right) \quad i, j \in \{1, 2\}, i \neq j \quad \text{(A-22)}$$

From these, the equilibrium prices and market shares are given by:

$$p_1 = \frac{1}{3}(v_1 - v_2) + t + \frac{1}{3}(q_1 + 2q_2)L \qquad p_2 = \frac{1}{3}(v_2 - v_1) + t + \frac{1}{3}(q_2 + 2q_1)L \tag{A-23}$$

$$n_1 = \frac{(v_1 - v_2) + 3t + (q_1 + 2q_2)L}{6t + 3(q_1 + q_2)L} \qquad n_2 = 1 - n_1 \tag{A-24}$$

In the first stage, firms select q_i to maximize $p_i n_i - c_i(q_i)$. For $i, j \in \{1, 2\}, i \neq j$, profit as a function of q_i, q_j is given by:

$$\pi_i(q_i, q_j) = \frac{((v_i - v_j) + 3t + (q_i + 2q_j)L)^2}{9(2t + (q_i + q_j)L)} - c_i(q_i) \tag{A-25}$$

Taking the derivative with respect to q_i yields:

$$\frac{d\pi_i(q_i, q_j)}{dq_i} = [v_j - v_i + t + q_i L] \left(\frac{L(v_i - v_j + 3t + (q_i + 2q_j)L)}{9(2t + (q_i + q_j)L)^2} \right) - c_i'(q_i) \tag{A-26}$$

The fraction term is strictly positive since $t > \frac{1}{3}(v_1 - v_2)$. Also, $c_i'(q_i) \leq 0$.

(i) For an equilibrium to satisfy $q_1 = q_2 = 0$, it must be the case that the derivative of each firm's profit function at $q_1 = q_2 = 0$ must be non-positive. Consider firm 2. The expression in the brackets becomes $[v_1 - v_2 + t] > 0$. Therefore, the derivative is positive.

(ii) For $q_i = 1$ to be a dominant strategy, the derivative of profit must be increasing for all q_i, q_j. This requires that the expression in the square brackets be positive, which is true whenever $t > v_1 - v_2$. □

Proof of Theorem 6. The consumer indifferent between Firms 1 and 2 ($u_1 = u_2$) is given by

$$\alpha_{12}^* = \frac{1}{\Delta}\left[(p_1 - p_2) + qL(n_1^e - n_2^e)\right] \tag{A-27}$$

By assumption, $n_1 + n_2 = 1$ and therefore $n_1 = 1 - \alpha_{12}^*$. In equilibrium, $n_j^e = n_j$, implying

$$n_1 = \frac{\Delta - (p_1 - p_2) + qL}{\Delta + 2qL} \tag{A-28}$$

Maximizing each firm's profit, $n_j p_j$, with respect to p_j and substituting yields the expressions in (12) and (13). As both p_2 and n_2 are increasing in q, the result holds for Firm 2. For Firm 1, profits are given by $p_1 n_1 = \frac{1}{9}\frac{(2\Delta + 3qL)^2}{\Delta + 2qL}$. Differentiating with respect to q yields the result. □