MIS Quarterly

RESEARCH ARTICLE

# MANAGING CONSUMER PRIVACY CONCERNS IN PERSONALIZATION: A STRATEGIC ANALYSIS OF PRIVACY PROTECTION

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# Appendix

## Equilibrium Pricing and Proofs

### *Equilibrium Pricing of the Personalized Products*

Cases 1, 2, and 3 can be represented and solved using a single general formulation. Let $\alpha$ and $\beta$ denote the proportion of those consumers who are willing to share personal information with firm A and B, respectively. There are different values for $\alpha$ ($\beta$), depending on firm A's (firm B's) protection choice; $\alpha = \beta = u$ in <No-Prot, No-Prot> case (Case 1); $\alpha = u + v$ and $\beta = u$ in <Prot, No-Prot> case (Case 2); $\alpha = \beta = u + v$ in <Prot, Prot> case (Case 3). Note that $\alpha \geq \beta$.

Given both firms' standard prices, $p_A$ and $p_B$, each firm sets personalized prices. First, we examine personalized pricing for the information-sharing consumers in the MP scopes (the $\alpha$ segment in the $MP_A$ scope and the $\beta$ segment in the $MP_B$ scope). Consider an information-sharing consumer located at $x$ in the $MP_i$ scope. This consumer is offered personalized products from firm $i$ and standard products from both firms. Because the net utility the consumer could get from the standard product offerings is $\max[R - p_A - tx, R - p_B - t(1-x)]$, she would choose firm $i$'s personalized product only when the net utility from the personalized product is at least $\max[R - p_A - tx, R - p_B - t(1-x)]$. Therefore, the price of the personalized product tailored to the consumer $x$, $q_i(x)$, should be $r(x)$ ($\equiv \min[p_A + tx, p_B + t(1-x)]$). Under this pricing, the consumer chooses the personalized product, and firm $i$'s profit from the consumer is maximized (in case of a tie in utility between a standard product and a personalized product, we assume that a consumer chooses the latter, because a firm can decrease the price of the personalized product infinitesimally, given the standard price).[1] Thus, the profits from the MP scopes are $\int_0^{1-s} \alpha r(x)dx$ for firm A and $\int_s^1 \beta r(x)dx$ for firm B.

[1]Although we assume the firms' commitment to personalization, it is easy to verify that firm $i$ is always better off by selling the personalized product to consumers in the $MP_i$ scope. Consider the $MP_A$ scope. First, for consumer $x$ such that $p_A + tx > p_B + t(1-x)$, firm $A$'s profit margin from its standard product is zero, because the consumer prefers the competitor's standard product. However, by offering a personalized product with $q_A(x) = p_B + t(1-x)$, firm $A$ can earn a profit of $p_B + t(1-x)$ from the consumer. Second, for consumer $x$ such that $p_A + tx < p_B + t(1-x)$, firm $A$'s profit margin from its standard product is $p_A$, which is dominated by the profit margin from its personalized product, $p_A + tx$. Thus, firm A always chooses to offer personalization. A similar argument is applied to firm B. Further, we can show that both firms offer personalization to the consumers in the CP scope.

Next, we examine personalized pricing for the consumers who are in the CP scope and share personal information with firm A (the $\alpha$ segment in the scope). Note that the $\alpha$ segment includes the $\beta$ segment since $\alpha \geq \beta$. In the following, we show that both firms' pricing for the personalized products does not exhibit a pure strategy equilibrium for $\alpha > \beta$, and derive a mixed strategy equilibrium. The solution for the case $\alpha = \beta$ is a degenerate case of the mixed strategy solution.

When $\alpha > \beta$ (i.e., the unconcerned and pragmatists share information with firm A, but only the unconcerned share information with firm B), the unconcerned are offered personalized products by both firms, while pragmatists are offered personalization by firm A only. However, since firm A cannot distinguish between the unconcerned and pragmatists on the same location, its pricing strategy should be the same for all consumers at $x$ in the $\alpha$ segment in the CP scope. Thus, firm A's pricing for the CP scope should consider the tradeoff between two conflicting objectives: (1) charging pragmatists as much as possible to extract the surplus from them and (2) attracting the unconcerned consumers (the $\beta$ segment) by undercutting firm B's price.

By focusing on the first objective, firm A can guarantee itself a profit of $(\alpha - \beta)r(x)$ from the consumers at $x$ by charging their effective reservation price $r(x)$. In this case, however, firm B can capture the entire $\beta$ segment at location $x$ with personalized products by undercutting firm A's price. Considering the second objective, firm A can set its price lower to get the $\beta$ segment at $x$, as long as the profit is larger than $(\alpha - \beta)r(x)$. However, firm B could react with a strategy of undercutting. This implies that if a firm fixes a price at each location, then its competitor can always undercut the price and capture the entire $\beta$ segment at the location. Therefore, there is no pure strategy equilibrium. Instead, we will have a mixed price equilibrium at each location inside the CP scope.

The construction of the mixed-strategy equilibrium at each point in the CP scope follows Varian (1980) and Narasimhan (1988). The consumers in the unconcerned-CP segment choose the firm that offers the lower price for the personalized products. Thus, the effect of the segment is similar to the informed and switching segments in Varian and in Narasimhan. The consumers in the pragmatists-CP segment always choose the personalized products offered by firm A. So, the effect of the segment is similar to the uninformed and loyal segments in Varian and in Narasimhan. Thus, from the technical perspective, the equilibrium at each point $x$ in our model is equivalent to the case in Narasimhan in which there are $v$ consumers who are loyal to firm A but not to firm B, there are $u$ consumers who are switchers, and the reservation price is $r(x)$, leading to the following results:[2]

(i) The equilibrium cumulative distribution function of the prices charged by firm $i$ for the personalized products targeted at $x$ in the CP scope, $F_{ix}(q)$ ($\equiv Pr(q_i(x) \geq q)$) is given by

$$F_{Ax}(q) = \begin{cases} 0, & q < \dfrac{\alpha-\beta}{\alpha} r(x), \\ 1 - \dfrac{\alpha-\beta}{\alpha}\dfrac{r(x)}{q}, & \dfrac{\alpha-\beta}{\alpha} r(x) \leq q < r(x), \\ 1, & q \geq r(x), \end{cases} \tag{1}$$

and

$$F_{Bx}(q) = \begin{cases} 0, & q < \dfrac{\alpha-\beta}{\alpha} r(x), \\ 1 - \dfrac{\alpha-\beta}{\beta}\dfrac{r(x)-q}{q}, & \dfrac{\alpha-\beta}{\alpha} r(x) \leq q < r(x), \\ 1, & q \geq r(x). \end{cases} \tag{2}$$

(ii) Firm A's expected profit from its personalized products targeted at $x$ in the CP scope is $(\alpha - \beta)r(x)$, and firm B's expected profit from its corresponding products is $(\beta/\alpha)(\alpha - \beta)r(x)$. ■

[2]Mixed pricing has been interpreted as a form of temporal sales (Narasimhan 1988; Varian 1980) or promotions with different discount level to different consumers (Shaffer and Zhang 2002). myShape.com also frequently offers targeted discount promotions with very short terms of validity (e.g., 24 hours) through e-mail.

## *Equilibrium Pricing of the Standard Products*

There are three distinct cases depending on the location of the fundamentalists who are indifferent in choosing between both firms' standard products. Let $y$ denote the location of the indifferent consumers. Then, $y = 1/2 + (p_B - p_A)/2t$. The first case is $y < 1-s$, the second case is $1-s \le y \le s$, and the third case is $y > s$. We find that only the second case is possible, as an equilibrium of the pricing game.

(i) **$y < 1 - s$ case:** In this case, the profit functions for both firms are

$$\pi_A = \alpha\left[\int_0^y (p_A + tx)dx + \int_y^{1-s}[p_B + t(1-x)]dx\right] + (\alpha-\beta)\int_{1-s}^{s}[p_B + t(1-x)]dx + (1-\alpha)\int_0^y p_A dx,$$
$$\pi_B = \beta\int_s^1 [p_B + t(1-x)]dx + \frac{\beta}{\alpha}(\alpha-\beta)\int_{1-s}^{s}[p_B + t(1-x)]dx + \left[(\alpha-\beta)\int_s^1 p_B dx + (1-\alpha)\int_y^1 p_B dx\right].$$

We obtain the following solution of the first-order conditions:

$$p_A = \frac{[3 + (4s-2)\beta]\alpha - (1+2s)\alpha^2 - 2(2s-1)\beta^2}{\alpha(1-\alpha)(3-2\alpha)}t,$$
$$p_B = p_A - \frac{(2s-1)(\alpha^2 - 2\alpha\beta + 2\beta^2)}{\alpha(3-2\alpha)}t.$$

With the above prices, $y = 1 - s + (2s-1)[3\alpha(1-\beta) + 2\beta(\alpha-\beta)]/2\alpha(3-2\alpha) > 1 - s$. Thus, the case condition is not satisfied, and no equilibrium exists for this case.

(ii) **$1 - s \le y \le s$ case**: Similarly to the above case, the profit functions for both firms in this case are

$$\pi_A = \alpha\int_0^{1-s}(p_A + tx)dx + (\alpha-\beta)\left[\int_{1-s}^{y}(p_A + tx)dx + \int_y^s [p_B + t(1-x)]dx\right] + (1-\alpha)\int_0^y p_A dx,$$
$$\pi_B = \beta\int_s^1 [p_B + t(1-x)]dx + \frac{\beta}{\alpha}(\alpha-\beta)\left[\int_{1-s}^{y}(p_A + tx)dx + \int_y^s [p_B + t(1-x)]dx\right]$$
$$+\left[(\alpha-\beta)\int_s^1 p_B dx + (1-\alpha)\int_y^1 p_B dx\right].$$

From the first-order conditions, we obtain the candidate equilibrium prices

$$p_A = \frac{(1-\beta)\gamma + 2(1-s)[(\alpha-\beta)^2 + \beta\gamma]}{(1-\alpha)[(\alpha-\beta)^2 + \gamma]}t,$$
$$p_B = p_A - \frac{(2s-1)(\alpha-\beta)^2}{(\alpha-\beta)^2 + \gamma}t,$$

where $\gamma = 3\alpha(1-\alpha) + 2\beta(\alpha-\beta)$.

Then, we have $y = 1 - s + (2s-1)[3\alpha(1-\alpha) + 2\beta(\alpha-\beta)]/2[3\alpha(1-\alpha) + \alpha^2 - \beta^2] = 1/2 - (2s-1)(\alpha-\beta)^2/2[3\alpha(1-\alpha) + \alpha^2 - \beta^2]$, which is between $1 - s$ and $1/2$. Thus, the case condition is always satisfied. With the above prices, the profits are

$$\pi_A = \frac{(2s-1)(2-\alpha-\beta)t}{4(1-\alpha)} \times \left[\begin{array}{l}\dfrac{(2s-1)(1-\alpha)(\alpha-\beta)^4}{[\gamma+(\alpha-\beta)^2]^2} - \dfrac{2(\alpha-\beta)^2[1+(2s-1)(\alpha-2\beta)]}{\gamma+(\alpha-\beta)^2} \\ +\dfrac{(1-\alpha+4\beta)[2-(1+2s^2)\alpha+(1+4s^2)\beta]+4s[\alpha(2-\alpha+4\beta)-\beta(5+4\beta)]}{(2s-1)(2-\alpha-\beta)}\end{array}\right],$$

$$\pi_B = \pi_A - \frac{(\alpha-\beta)t}{4\alpha} \times \left[ \begin{array}{l} \alpha-\beta-2(1-s)^2(\alpha-2\beta)+\dfrac{4(2s-1)(\alpha-\beta)[1-(2s-1)\beta]}{1-\alpha}+\dfrac{(2s-1)^2[2\gamma+3(\alpha-\beta)^2](\alpha-\beta)^3}{[\gamma+(\alpha-\beta)^2]^2} \\ +\dfrac{2(2s-1)(\alpha-\beta)\left[2(2s-1)(\alpha^3+\beta^3)+[5+2\beta-4s(2+\beta)]\alpha^2-3(2s-1)\beta^2-\alpha[1-(2s-1)(8-3\beta)\beta]\right]}{(1-\alpha)[\gamma+(\alpha-\beta)^2]} \end{array} \right],$$

Next, we need to examine each firm's unilateral deviation incentive from the strategies in this candidate equilibrium. There are four possible deviation scenarios. Firm A can deviate by increasing its standard price in such a way that the resulting $y$ is less than $1 - s$ (deviation to $y < 1 - s$ case) or by decreasing the standard price in such a way that the resulting $y$ is larger than $s$ (deviation to $y > s$ case below). Similarly firm B can deviate to $y < 1 - s$ case by decreasing its standard price, or to $y > s$ case by increasing its standard price. It is easily verified that firms do not have deviation incentives for $\alpha = \beta$ (Cases 1 and 3). It is also found that when $\alpha > \beta$ (Case 2), only the second scenario (firm A's deviation to $y > s$ case) can be profitable when $s$ is sufficiently small *and* $\alpha$ is sufficiently high, while firms do not have deviation incentives in the other three scenarios.

The equilibrium prices and profits for Cases 1, 2, and 3 can be obtained by replacing $\alpha$, $1 - \alpha$, and $\beta$ with the corresponding values for the cases as follows:

$$p_1^* = p_{1A}^* = p_{1B}^* = t\left[1+\frac{2(1-s)u}{v+w}\right],$$

$$\pi_1^* = \pi_{1A}^* = \pi_{2B}^* = \frac{t}{2}\left[1-(3s-2)su+\frac{4(1-s)^2u}{1-u}\right], \tag{3}$$

$$p_{2A}^* = \frac{(v+w)\gamma_2+2(1-s)(v^2+u\gamma_2)}{w(v^2+\gamma_2)}t \quad \text{and} \quad p_{2B}^* = p_{2A}^* - \frac{(2s-1)v^2}{v^2+\gamma_2}t, \tag{4}$$

where $\gamma_2 = 2uv + 3(u + v)w$,

$$\pi_{2A}^* = \frac{(2s-1)(1-u+w)t}{4w} \times \left[ \begin{array}{l} \dfrac{[2-v+2s^2(u-v)](4u+w)-4s[u(4u+3w)-v(1+w)]}{(2s-1)(1-u+w)} \\ -\dfrac{2[1-(2s-1)(u-v)]v^2}{v^2+\gamma_2}+\dfrac{(2s-1)wv^4}{(v^2+\gamma_2)^2} \end{array} \right],$$

$$\pi_{2B}^* = \pi_{2A}^* - \frac{t}{4(u+v)} \times \left[ \begin{array}{l} 2(1-s)^2(u-v)v+v^2+4(2s-1)[1-(2s-1)u]\dfrac{v^2}{w}+(3v^2+2\gamma_2)\dfrac{(2s-1)^2v^4}{(v^2+\gamma_2)^2} \\ +\left[(1-w)w[(2s-1)(u-v)-1]+[4(1-s)(1-u)-(2s+1)w]v^2-2v^3\right]\dfrac{2(2s-1)v^2}{w(v^2+\gamma_2)} \end{array} \right], \tag{5}$$

$$p_3^* = p_{3A}^* = p_{3B}^* = \left[1+\frac{2(1-s)(u+v)}{w}\right]t,$$

and

$$\pi_3^* = \pi_{3A}^* = \pi_{3B}^* = \frac{t}{2}\left[1-(3s-2)s(u+v)+\frac{4(1-s)^2(u+v)}{1-u-v}\right].$$

(iii) **$y > s$ case:** In this case, both firms' profit functions are

$$\pi_A = \int_0^{1-s} \alpha r(x)\,dx + \int_{1-s}^{s} (\alpha-\beta) r(x)dx + \int_s^y (\alpha-\beta) p_A dx + \int_0^y (1-\alpha) p_A dx,$$
$$\pi_B = \int_s^1 \beta r(x)dx + \int_{1-s}^{s} \frac{\beta}{\alpha}(\alpha-\beta) r(x)dx + \left[\int_y^1 (\alpha-\beta) p_B dx + \int_y^1 (1-\alpha) p_B dx\right].$$

After replacing $r(x)$ and then solving the first-order conditions simultaneously, we have

$$p_A = \frac{3-4s\beta+(2s-1)\beta^2}{(1-\beta)(3-2\beta)} t,$$
$$p_B = p_A + \frac{(2s-1)\beta}{(3-2\beta)} t.$$

With the above prices, $y = s - 3(2s-1)(1-\beta)/(6-4\beta) < s$. Thus, the case condition is not satisfied, and no equilibrium exists for this case. ■

## *Proof of Proposition 1*

**Part (i):** Since $p_{2A}^* > p_{2B}^*$ from equation (4), it is sufficient to show that $p_{2B}^* > p_1^*$.

$$p_{2B}^* - p_1^* = \frac{(v+w)(2uv+3uw+vw) + 2(1-s)[v^2 + u\gamma + 2vw(v+w)]}{w(v+w)(v^2+\gamma)} vt > 0$$

Thus, $p_{2B}^* > p_1^*$. So, the consumers in the fundamentalist segment and the pragmatists in the $MP_B$ scope, who buy the standard products, are charged higher standard prices by both firms in Case 2 compared with those in Case 1.

Further, it is trivial to verify that $p_{2A}^*$ and $p_{2B}^*$ are increasing in $v$.

**Parts (ii) and (iii):** Consider the unconcerned and pragmatists located at $x$ in firm A's MP scope. They are charged $p_{2A}^* + tx$ and buy the firm's personalized product in Case 2. Since $p_{2A}^* + tx > p_1^* + tx \geq p_1^*$, the consumers are charged higher prices than in Case 1. Similarly, the unconcerned in firm B's MP scope are also charged higher prices for the firm's personalized products than in Case 1, since $p_{2B}^* + tx > p_1^* + tx$.

Next, consider the unconcerned in the CP scope, who buy personalized products in both cases. Let $APP_{2i}^{CP}$ denote the average personalized price charged by firm $i$ for the CP segment in Case $k$. Then, from (1) and (2), it is easy to see that $APP_{2i}^{CP} > APP_1^{CP} = 0$. Thus, both firms charge higher on average for the personalized products for the segment in Case 2 than in Case 1.

Finally, consider the pragmatists in the CP scope, who buy standard products in Case 1 but buy firm A's personalized products in Case 2. Let $f_{Ax}(q)$ denote the probability density function of $q_A(x)$. From (1), we have $f_{Ax}(q) = vr(x)/(u+v)q^2$ and $F_{Ax}(q)$ has a probability mass point at $r(x)$ equal to $v/(u+v)$. Thus, the average value of $q_A(x)$, $E[q_A(x)]$, is calculated as follows:

$$E[q_A(x)] = \int_{vr(x)/(u+v)}^{r(x)} q f_{Ax}(q)dq + \frac{vr(x)}{u+v} = \frac{vr(x)}{u+v}\left[1 + \log(\frac{u+v}{v})\right].$$

Then $APP_{2A}^{CP} = \dfrac{1}{1-2s}\displaystyle\int_{1-s}^{s} E[q_A(x)]dx$.

From Equation (3), it is easily verified that $p_1^*$ is decreasing in $v$ and approaches $t$ as $v$ approaches 1. Since both $p_{2A}^*$ and $p_{2B}^*$ increasing in $v$, $E[q_A(x)]$ and thus $APP_{2A}^{CP}$ increases with $v$. $E[q_A(x)]$ and $APP_{2A}^{CP}$ approaches 0 as $v$ approaches 0. Further, $E[q_A(x)]$ and $APP_{2A}^{CP}$ approaches $r(x)$ as $v$ approaches 1. Therefore, $APP_{2A}^{CP} - p_1^*$ is negative when $v$ approaches 0, increases as $v$ increases, and is positive when $v$ approaches 1.

**Part (iv)**: This part of the proof is included in the previous parts.

| | | Firm B | |
|---|---|---|---|
| | | No-Prot | Prot |
| Firm A | No-Prot | $\Pi_1^*$, $\Pi_1^*$ | |
| | Prot | $\Pi_{2A}^*$, $\Pi_{2B}^*$ | $\Pi_3^*$, $\Pi_3^*$ |

**Figure A1. The First Stage of the Game**

## *Proof of Proposition 2*

Let $\Pi_{ki}^*$ denote the equilibrium net profit of firm $i$ in Case $k$. Then, $\Pi_1^* = \Pi_{1A}^* = \Pi_{1B}^* = \pi_1^* - K_I^u$, $\Pi_{2A}^* = \pi_{2A}^* - K_P - K_I^{u+v}$, $\Pi_{2B}^* = \pi_{2B}^* - K_I^u$, and $\Pi_3^* = \Pi_{3A}^* = \Pi_{3B}^* = \pi_3^* - K_P - K_I^{u+v}$. Figure A1 shows the payoff matrix for the first stage of the game.

<No-Prot, No-Prot> is the equilibrium if and only if neither firm can deviate profitably by adopting privacy protection. This condition is equivalent to $\Pi_1^* > \Pi_{3A}^*$, which is reduced to $K > \pi_{2A}^* - \pi_1^*$.

<Prot, Prot> is the equilibrium if and only if neither firm has an incentive of unilateral deviation by not adopting privacy protection. This condition is equivalent to $\Pi_3^* > \Pi_{2B}^*$, which is reduced to $K < \pi_3^* - \pi_{2B}^*$. It can be shown that $\pi_3^* - \pi_{2B}^*$ is a quadratic and convex function of $s$. Further, it can be shown that $\pi_3^* - \pi_{2B}^* > 0$ at $s = 1/2$ and $\pi_3^* - \pi_{2B}^* < 0$ at $s = 1$. Thus, $\pi_3^* - \pi_{2B}^* > 0$ for $s < s'(u, v, w)$ and $\pi_3^* - \pi_{2B}^* < 0$ for $s > s'(u, v, w)$, where $s'(u, v, w)$ is the smaller solution of the equation $\pi_3^* - \pi_{2B}^* = 0$. Therefore, <Prot, Prot> is the equilibrium for $s < s'(u, v, w)$ and $K < \pi_3^* - \pi_{2B}^*$. On the other hand, <Prot, Prot> cannot be an equilibrium for $s > s'(u, v, w)$.

Finally, <Prot, No-Prot> is the equilibrium if and only if $\Pi_1^* < \Pi_{3A}^*$ and $\Pi_3^* < \Pi_{2B}^*$. From the above results, these two conditions are equivalent to $s > s'(u, v, w)$ and $K < \pi_3^* - \pi_{2B}^*$, or $\pi_3^* - \pi_{2B}^* < K < \pi_{2A}^* = \pi_1^*$. ■

## *Proof of Corollary 1*

Part (i) of the proposition is equivalent to $\partial s'(u, v, w)/\partial w > 0$. Because of the complex nature of the formula, we need to rely on numerical procedures on a dense grid of $u$, $v$, and $w$ to check the sign of $\partial s'(u, v, w)/\partial w$. Specifically, we let $u$, $v$, and $w$ vary from 0.01 to 0.99 by 0.01 and ensure that the constraint $u + v + w = 1$. We find that the sign is invariably positive.

Part (ii) of the proposition can be verified by showing that for any given value of $w$, $s'(u, v, w)$ increases with $v/u$. Again we apply numerical analysis. First, we vary $w$ from 0.01 to 0.99 by 0.01. Then, for each value of $w$, we vary $v$ from 0.01 to 0.99 – $w$ by 0.01 and set $u = 1 - v - w$, which ensures $v/u$ increases for the given $w$. We find that $s'(u, v, w)$ invariably increases with $v/u$ for any given value of $w$. ■

## *Proof of Proposition 3*

The social welfare when <No-Prot, No-Prot> is the equilibrium ($SW_1$) is

$$SW_1 = uR + (v+w)\left[\int_0^{1/2}(R-tx)dx + \int_{1/2}^{1}[R-t(1-x)]dx\right] - 2K_I^u$$

$$= R - \frac{(v+w)t}{4} - 2K_I^u.$$

The social welfare when <Prot, No-Prot> is the equilibrium ($SW_2$) is

$$SW_2 = uR + v\left[sR + \int_s^1 [R - t(1-x)]dx\right] + w\left[\int_0^y (R - tx)dx + \int_y^1 [R - t(1-x)]dx\right] - (K_P + K_I^{u+v} + K_I^u)$$
$$= R - \frac{wt}{4} - \frac{(1-s)^2 vt}{2} - \frac{(2s-1)^2 v^4 wt}{4[3u(1-u) + v(3-2v) - 4uv]^2} - (K_P + K_I^{u+v} + K_I^u).$$

Then social welfare when <Prot, Prot> is the equilibrium ($SW_3$) is

$$SW_3 = (u+v)R + w\left[\int_0^{1/2} (R - tx)dx + \int_{1/2}^1 [R - t(1-x)]dx\right] - 2(K_P + K_I^u)$$
$$= R - \frac{wt}{4} - 2(K_P + K_I^u).$$

**Part (i)**: $SW_2 - SW_1$ is calculated as follows:

$$SW_2 - SW_1 = \frac{[1 - 2(1-s)^2]vt}{4} - \frac{(2s-1)^2 v^4 wt}{4[3u(1-u) + v(3-2v) - 4uv]^2} - K \equiv L - K.$$

Thus, $SW_2 - SW_1 > 0$ for $K < L$. Note from Proposition 2 that a necessary condition for <Prot, No-Prot> being the equilibrium is $K < \pi_{2A}^* - \pi_1^*$. By algebraic manipulations, we find that $L < \pi_{2A}^* - \pi_1^*$. So, the parameter set that satisfies $K < L$ is a subset of the parameter set that satisfies $K < \pi_{2A}^* - \pi_1^*$.

It can be verified that $L = 0$ at $v = 0$, $L = [1 - 2(1-s)^2]t/4$ at $v = 1$, and $L$ is monotonically increasing in $v$. Thus, when $K < [1 - 2(1-s)^2]t/4$, social welfare increases with the protection for $v > v'$, and decreases for $v < v'$, where $v$ solves the equation $K = L$. When $K > [1 - 2(1-s)^2]t/4$, social welfare always decreases with protection.

Next, let's consider the consumer welfare ($CW$). $CW_2 - CW_1 = (SW_2 - \Pi_{2A}^* - \Pi_{2B}^*) - (SW_1 - 2\,\Pi_1^*)$ can be shown to be a quadratic function of $s$ and negative both at $s = 1/2$ and at $s = 1$. Thus, when $CW_2 - CW_1$ is convex in $s$, it is always negative. When $CW_2 - CW_1$ is concave in $s$, it can be shown that $CW_2 - CW_1$ is always increasing in $s$ for $1/2 < s < 1$. Thus, consumer welfare always decreases with protection.

**Part (ii)**: $SW_3 - SW_1 = vt/4 - 2K$. Thus, $SW_3 - SW_1 > 0$ for $K < vt/8$. From Proposition 2, <Prot, Prot> can be the equilibrium choice only if $K < \pi_3^* - \pi_{2B}^*$. We find that $vt/8 > \pi_3^* - \pi_{2B}^*$ as follows. First, $vt/8 - (\pi_3^* - \pi_{2B}^*)$ is quadratic and concave in $s$. Next, it is found that $vt/8 - (\pi_3^* - \pi_{2B}^*) = 0$ at $s = 1/2$, and $vt/8 - (\pi_3^* - \pi_{2B}^*) > 0$ at $s = 1$. Thus, we always have $vt/8 - (\pi_3^* - \pi_{2B}^*) > 0$. So, whenever <Prot, Prot> is the equilibrium choice, $K < vt/8$ is always satisfied. Thus, social welfare always increases with protection.

Next, $CW_3 - CW_1 = (SW_3 - SW_1) - 2(\Pi_3^* - \Pi_1^*)$ is calculated as follows:

$$CW_3 - CW_1 = \frac{(1 - 8s + 12s^2)vt}{4} - \frac{4(1-s)^2 vt}{(1-u)(1-u-v)}.$$

By solving $CW_3 - CW_1 > 0$, we have

$$s > \frac{3 + 2u - u^2}{1 + 6u - 3u^2} - \frac{1}{6}\sqrt{1 + \frac{4(47 - 102u + 51u^2)}{(1 + 6u - 3u^2)^2}} \equiv s'' \text{ and } v < 1 - u - \frac{16(1-s)^2}{(1-u)(1 - 8s + 12s^2)} \equiv v''.$$

However, it can be shown that $s'' > s'$. Since <Prot, Prot> can be the equilibrium choice only if $s < s'$, the condition $s > s''$ cannot be satisfied. Thus, consumer welfare always decreases with protection. ■

**Proof of Proposition 4**: **Part (i)**: From the proof of Part (ii) of Proposition 3, $SW_3 - SW_1 > 0$ for $K < vt/8$. From Proposition 2, <No-Prot, No-Prot> is the equilibrium choice if and only if $K > \pi_{2A}^* - \pi_1^*$. However, we find that $vt/8 < \pi_{2A}^* - \pi_1^*$, and thus $K < vt/8$ cannot be satisfied under the <No-Prot, No-Prot> equilibrium choice, as follows. First, $(\pi_{2A}^* - \pi_1^*) - vt/8$ is found to be quadratic in $s$, and positive both at $s = 1/2$ and $s = 1$. Second, when $(\pi_{2A}^* - \pi_1^*) - vt/8$ is convex, it can be shown that $(\pi_{2A}^* - \pi_1^*) - vt/8$ is always decreasing in $s$ between 1/2 and 1. So, we always have $vt/8 < \pi_{2A}^* - \pi_1^*$. Thus, social welfare always decreases with regulation.

Next, from the proof of Part (ii) of Proposition 3, $CW_3 - CW_1 > 0$ for $s > s''$ and $v < v''$.

**Part (ii):** $SW_3 - SW_2$ is calculated as follows:

$$SW_3 - SW_2 = \frac{(1-s)^2 vt}{2} + \frac{(2s-1)^2 v^4 wt}{4[3u(1-u)+v(3-2v)-4uv]^2} - K \equiv M - K.$$

Thus, $SW_3 - SW_2 > 0$ for $K < M$. Note from Proposition 2 that a necessary condition for <Prot, No-Prot> being the equilibrium choice is $K < \pi^*_{2A} - \pi^*_1$. By algebraic manipulations, we find that $M < \pi^*_{2A} - \pi^*_1$. So, the parameter set that satisfies $K < M$ is a subset of the parameter set that satisfies $K < \pi^*_{2A} - \pi^*_1$.

It can be verified that $M = 0$ at $v = 0$, $M = (1 - s)^2 t/2$ at $v = 1$, and $M$ is monotonically increasing in $v$. Thus, when $K < (1 - s)^2 t/2$, social welfare increases with the regulation for $v > v^+$, and decreases for $v < v^+$, where $v^+$ solves the equation $K = M$. When $K > (1 - s)^2 t/2$, social welfare always decreases with regulation.

Next, $CW_3 - CW_2 = (SW_3 - 2\Pi^*_3) - (SW_2 - \Pi^*_{2A} - \Pi^*_{2B})$ can be shown to be quadratic and concave in $s$. Further, it can be verified that $CW_3 - CW_2 = 0$ at $s = 1/2$ and $CW_3 - CW_2 > 0$ at $s = 1$. Thus, $CW_2 - CW_1$ is always positive for $1/2 < s < 1$; that is, consumer welfare increases with regulation. ■

## References


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