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RESEARCH ARTICLE

COMPLEMENTARY ONLINE SERVICES IN COMPETITIVE MARKETS: MAINTAINING PROFITABILITY IN THE PRESENCE OF NETWORK EFFECTS

Hila Etzion

Department of Technology and Operations, Stephen M. Ross School of Business, University of Michigan, Ann Arbor, MI 48103 U.S.A. {etzionh@umich.edu}

Min-Seok Pang

Department of Information Systems and Operations Management, School of Management, George Mason University, Fairfax, VA 22030 U.S.A. {mpang2@gmu.edu}

Appendix A

Derivations of Equilibrium Prices and Profits per Market Configuration

Configuration

We derive the equilibrium in prices and demands given the choices of the two firms in the first stage of the game. We consider only cases in which (1) each firm has positive demand for its product, and (2) market is covered. The required conditions on the parameters values are given in the following assumption.

Assumption 1. *Conditions for Spatial Competition in Equilibrium*

(i) $2t > M(\alpha_A + \alpha_B)$, where $\alpha_i = 0$ when firm i does not offer service (A1)

(ii) $c - 3t + \alpha_j M - \Delta_i < s_i < 3t - 2\alpha_i M + c - \Delta_i$ ($i, j = A$ and B, $i \neq j$) (A2)

(iii) $-3t + M(\alpha_i + 2\alpha_i) - \Delta_i < s_i - s_j < 3t - M(2\alpha_i + \alpha_j) - \Delta_i$ (A3)

(iv) $|m_i - m_j| < 3t$

(v) $V > \max\left(\frac{3t+m_A+m_B}{2}, V_{SN}, V_{NS}, V_{SS}\right)$ (A4)

Assumption 1-(ii) ensures that an equilibrium in which both firms have positive demand prevails when only one of the firms offers the service (else, one firm would set a price to undercut the other and capture the entire market). Similarly, Assumption 1-(iii) ensures both firms have positive demand when both firms offer the service, and Assumption 1-(iv) ensures both firms have positive demand when both sell only the product. Assumption 1-(i) is necessary for the ranges given in Assumptions 1-(ii) and (iii) to be none empty, and is thus implied by the other two conditions. An identical assumption is set in Li and Chen (2012) (where $M = 1$ and θ denote the degree of network effects), who state: "If $t < \theta$, the network effects dominate employees' preferences over product's stand-alone value and employees will always purchase form one single seller." As is shown below, given condition (i), all S.O.C are satisfied.

Finally, Assumption 1-(iv) ensures that the inherent value of the product, V, is sufficiently high so that the market for the product is covered by the two firms, whether both, neither, or only one firm offer the service.

1. Case NN: Both Firms Sell Only Product

When neither firm operates the service, the surplus a consumer obtains when buying the product sold by Firm A and the surplus from buying the product sold by Firm B, are given respectively by

$$u^A_{NN} = V - tx - p^A_{NN} \tag{A5}$$

$$u^B_{NN} = V - t(1-x) - p^B_{NN} \tag{A6}$$

For spatial competition (the market is covered and the marginal customer has positive utility), it must be that $V > \frac{3t+m_A+m_B}{2}$. It is easy to show that when this condition holds, in equilibrium th product price is

$$p^{i}_{NN}{}^{*} = \frac{3t+2m_i+m_j}{3}\left(i, j = A \text{ and } B, i \neq j\right) \tag{A7}$$

The market share of Firm i is $\frac{M(3t-m_i+m_j)}{6t}$, and its profit is given by

$$\pi^{i}_{NN}{}^{*} = \frac{M(3y-m_i+m_j)^2}{18t} \tag{A8}$$

In this paper, we limit our attention to cases of spatial competition; that is, we assume $V > \frac{3t+m_A+m_B}{2}$ (see Assumption 1-(iv)).

2. Cases SN and NS: Only One Firm Offers a Service

Without loss of generality, we assume that only Firm A decided to offer a service to its customers. The solution when only Firm B offers the service can be derived in a similar manner.

When expected network size of firm A is N_A, the consumer surplus when buying from Firm A, u^A_{SN}, and when buying from Firm B, u^B_{SN} are given by

$$u^A_{SN} = V - tx + s_A N_A - p^A_{SN} \tag{A9}$$

$$u^B_{SN} = V - t(1-x) - p^B_{SN} \tag{A10}$$

The location of the customer who is indifferent between the two firms, denoted by $\hat{x}$, is thus

$$\hat{x}(N_A) = \frac{t+\alpha_A N_A + s_A - p^A_{SN} + p^B_{SN}}{2t} \tag{A11}$$

The demand for the product and service of Firm A, D^A_{SN}, given that consumers expect the number of service users to be N_A, is given by $M\hat{x}(N_A)$. In the fulfilled expectation equilibrium, we require that

$$D^A_{SN} = M\hat{x}(D^A_{SN}) \tag{A12}$$

Solving the above equation for D^A_{SN}, we get

$$D^A_{SN} = \frac{M\left(t+s_A-p^A_{SN}+p^B_{SN}\right)}{2t-\alpha_A M} \tag{A13}$$

Given our assumption that the market is covered, the demand for Product B is given by $D^B_{SN} = M - D^A_{SN}$. Finally, the profit functions of two firms are given by $\pi^A_{SN} = D^A_{SN}(p^A_{SN} - m_A - c)$ and $\pi^B_{N} = D^B_{SN}(p^B_{SN} - m_B)$.

Solving the first-order conditions simultaneously (S.O.C requires $2t > \alpha_A M$, which is satisfied due to Assumption 1-(i)), we find that in equilibrium prices and profits are as follows:

$$p_{SN}^{A\ *} = \frac{3t-\alpha_A M+s_A+2c+2m_A+m_B}{3}, \quad p_{SN}^{B\ *} = \frac{3t-2\alpha_A M-s_A+c+m_A+2m_B}{3} \tag{A14}$$

$$\pi_{SN}^{A\ *} = \frac{M(3t-c-\alpha_A M+s_A-m_A+m_B)^2}{9(2t-\alpha_A M)}, \quad \pi_{SN}^{B\ *} = \frac{M(3t+c-2\alpha_A M-s_A+m_A+m_B)^2}{9(2t-\alpha_A M)} \tag{A15}$$

At the above prices, the condition for both firms to have positive demand (i.e., $0 < D_{SN}^i < M$ for $i = A, B$) is

$$c - 3t + \alpha_A M - m_A + m_B < s_A < 3t - 2\alpha_A M + c - m_A + m_B \tag{A16}$$

To ensure spatial competition at the above prices, we need to find the surplus of the customer indifferent between the two products and require it to be positive. Doing so we get the following condition:

$$V > V_{SN} = \frac{(3t-\alpha_A M)(3t+c-2\alpha_A M-s_A+m_A-m_B)-\alpha_A m_B M}{6t-3\alpha_A M} \tag{A17}$$

3. Case SS: Both Firms Offer a Service

When both firms offer the service, the utility functions are given by

$$u_{SS}^A = V - tx + s_A N_A - p_{SS}^A \tag{A18}$$

$$u_{SS}^B = V - t(1-x) + s_B + \alpha_B N_B - p_{SS}^B \tag{A19}$$

The location of indifferent customer $\hat{x}$ is found by solving $u_{SS}^A = u_{SS}^B$ and is given by

$$\hat{x}(N_A, N_B) = \frac{t+s_A-s_B+\alpha_A N_A-\alpha_B N_B-p_{SS}^A+p_{SS}^B}{2t} \tag{A20}$$

The demand for the product and service of Firm A, D_{SS}^A, given consumers expectations regarding network sizes, is $M\hat{x}(N_A, N_B)$, and the demand for the product and service of Firm B, D_{SS}^B, given the assumption that the market is covered is $M - D_{SS}^A$. In the fulfilled expectation equilibrium, we require that

$$D_{SS}^A = M\hat{x}(D_{SS}^A, D_{SS}^B) \quad \text{and} \quad D_{SS}^B = M(1 - \hat{x}(D_{SS}^A, D_{SS}^B)) \tag{A21}$$

Solving the above two equations simultaneously for D_{SS}^A and D_{SS}^B, we get

$$D_{SS}^A = \frac{M(t-\alpha_A M+s_A-s_B-p_{SS}^A+p_{SS}^B)}{2t-M(\alpha_A+\alpha_B)}, \quad D_{SS}^B = \frac{N(t-\alpha_B N+s_B-s_A-p_{SS}^A+p_{SS}^B)}{2t-M(\alpha_A+\alpha_B)} \tag{A22}$$

The profit functions of the two firms are given by

$$\pi_{SS}^i = D_{SS}^i(p_{SS}^i - m_i - c) \ (i = A \text{ and } B) \tag{A23}$$

Solving the first order conditions simultaneously (second order condition requires $2t > (\alpha_A + \alpha_B)M$, which is satisfied according to Assumption 1-(i)), we find the equilibrium prices

$$\begin{aligned} p_{SS}^A &= \tfrac{1}{3}(s_A - s_B + 3c + 3t - M(\alpha_A + 2\alpha_B) + 2m_A + m_B) \\ p_{SS}^B &= \tfrac{1}{3}(s_B - s_A + 3c + 3t - M(2\alpha_A + \alpha_B) + m_A + 2m_B) \end{aligned} \tag{A24}$$

The profits at the optimal prices are given by

$$\pi_{SS}^{A} = \frac{M\left(3t - M(\alpha_A + 2\alpha_B) + s_A - s_B - m_A + m_B\right)^2}{9\left(2t - M(\alpha_A + \alpha_B)\right)}, \quad \pi_{SS}^{B} = \frac{M\left(3t - M(2\alpha_A + \alpha_B) + s_B - s_A + m_A - m_B\right)^2}{9\left(2t - M(\alpha_A + \alpha_B)\right)} \tag{A25}$$

The condition for both firms to have positive demand (i.e., the marginal customer's location is interior) is

$$-3t + M(\alpha_A + 2\alpha_B) + m_A - m_B < s_A - s_B < 3t - M(2\alpha_A + \alpha_B) + m_A - m_B \tag{A26}$$

which also requires that

$$2t > M(\alpha_A + \alpha_B) \tag{A27}$$

or else above range for $s_A - s_B$ values is empty. Finally, with the above prices, there is spatial completion if and only if

$$V > V_{SS} = C - \tfrac{1}{3}\left(2s_A + s_B - 2m_A - m_B - 5t + M\left(3\alpha_A + 2\alpha_B\right)\right) + \frac{(t - \alpha_A M)(s_B - s_A + t - \alpha_A M + m_A - m_B)}{2t - M(\alpha_A + \alpha_B)} \tag{A28}$$

Appendix B

Proofs

Proof of Proposition 1

Having obtained the equilibrium prices and profits in Appendix A (see also Tables 2 and 3 in the paper), we now derive the conditions for each possible market configuration to be an equilibrium. The conditions are derived as follows:

(i) Both firms offer the service in equilibrium if and only if $\pi_{SS}^{A} > \pi_{NS}^{A}$ and $\pi_{SS}^{B} > \pi_{SN}^{B}$.

(ii) Both firms sell only product in equilibrium if and only if $\pi_{NN}^{A} > \pi_{SN}^{A}$ and $\pi_{NN}^{B} > \pi_{NS}^{B}$.

(iii) Only Firm A offers a service in equilibrium if and only if $\pi_{SN}^{A} > \pi_{NN}^{A}$ and $\pi_{SN}^{B} > \pi_{SS}^{B}$.

(iv) Only Firm B offers a service in equilibrium if and only if $\pi_{NS}^{A} > \pi_{SS}^{A}$ and $\pi_{NS}^{B} > \pi_{NN}^{B}$.

Equilibrium in Which Both Firms Sell the Service

In order for both Firm A and Firm B to offer the service in equilibrium, it must be that $\pi_{SS}^{A} > \pi_{NS}^{A}$ and $\pi_{SS}^{B} > \pi_{SN}^{B}$, so that neither firm has incentive to deviate and not sell the service. These two conditions are given by

$$s_i > X_i s_j + Y_i \text{ for } (i = A\ , j = B) \text{ and for } (i = B, j = A)$$

where $X_i = 1 - \frac{\sqrt{2t - M(\alpha_A + \alpha_B)}}{\sqrt{2t - \alpha_j M}}$

and $Y_i = \frac{\left(3t - 2\alpha_j M + c + m_i - m_j\right)\sqrt{2t - M(\alpha_A + \alpha_B)}}{\sqrt{2t - \alpha_j M}} - \left(3t - M\left(\alpha_A + \alpha_B + \alpha_j\right) - m_i + m_j\right)$.

Equilibrium in Which Neither Firm Sells the Service

An equilibrium in which neither firm provides the service exists if and only if $\pi_{NN}^{A} > \pi_{SN}^{A}$ and $\pi_{NN}^{B} > \pi_{NS}^{B}$, so that neither firm has incentive to deviate and offer the service. From the profit expressions in Table 3, we find that $\pi_{NN}^{A} > \pi_{SN}^{A}$ if and only if

$$s_A < \frac{(3t-m_i+m_j)\sqrt{2(2t-\alpha_i M)}}{2\sqrt{t}} - 3t + \alpha_i M + c + m_i - m_i \tag{A29}$$

We denote this upper bound by $\bar{s}_A$. $\bar{s}_B$ can be derived in a similar manner.

Equilibrium in Which Only Firm A Sells the Service

The conditions under which there is an equilibrium in which only Firm A offers the service are (i) $\pi_{SN}^{B} > \pi_{SS}^{B}$ and (ii) $\pi_{SN}^{A} > \pi_{NN}^{A}$. Condition (i) implies that Firm B does not have an incentive to deviate and offer the service. Condition (ii) indicates that Firm A does not have an incentive to deviate and not offer the service. Condition (i) and (ii) translate to $s_B < X_B s_A + Y_B$ and $s_A > \bar{s}_A$, respectively. The conditions under which an equilibrium in which only Firm B sells the service is feasible can be derived in a similar manner.

Proof of Proposition 2

We derive the condition for $\pi_{SS}^{i} < \pi_{NN}^{i}$.

$\pi_{SS}^{i} = \frac{M(3t-M(\alpha_i+2\alpha_j)+s_i-s_j-m_i+m_j)^2}{9(2t-M(\alpha_i+\alpha_j))}$, and given Assumption 1-(iii), we have $3t - M(\alpha_i + 2\alpha_j) + s_i - s_j + m_i - m_j > 0$. Thus $\pi_{SS}^{i} = \frac{M(3t-M(\alpha_i+2\alpha_j)+s_i-s_j-m_i+m_j)^2}{9(2t-M(\alpha_i+\alpha_j))} < \pi_{NN}^{i} = \frac{M(3t-m_i+m_j)^2}{18t}$ If and only if

$$\left(3t - M(\alpha_i + 2\alpha_i) + s_i - s_j - m_i + m_j\right) < \frac{(3t-m_i+m_j)\sqrt{2(2t-M(\alpha_i+\alpha_j))}}{2\sqrt{t}} \tag{A30}$$

Rearranging terms, we get

$$s_i - s_j < M(\alpha_i + 2\alpha_j) + s_i - s_j + m_i - m_j + \frac{(3t-m_i+m_j)\sqrt{(2t-M(\alpha_i+\alpha_j))}}{\sqrt{2t}}$$

Proof of Proposition 3

(i) We examine the derivative of the profit of Firm A, when both firms offer the service, with respect to the α_A.

$$\begin{aligned}
\frac{\partial}{\partial\alpha_A}\pi_{SS}^{A} &= \frac{\partial}{\partial\alpha_A}\frac{M(3t-M(\alpha_A+2\alpha_B)+s_A-s_B-m_A+m_B)^2}{9(2t-M(\alpha_A+\alpha_B))^2} \\
&= -\frac{M^2(t-\alpha_A M-s_A+s_B+m_A-m_B)(3t-M(\alpha_A+2\alpha_B)+s_A-s_B-m_A+m_B)}{9(2t-M(\alpha_A+\alpha_B))^2} \\
&= -D_{SS}^{A}\,\frac{M(t-\alpha_A M-s_A+s_B+m_A-m_B)}{3(2t-M(\alpha_A+\alpha_B))}
\end{aligned} \tag{A31}$$

The above is positive if and only if $\frac{M(t-\alpha_A M-s_A+s_B+m_A-m_B)}{3(2t-M(\alpha_A+\alpha_B))}$ is negative, which is equivalent to $s_A > s_B + t - \alpha_A M + m_A - m_B$. This is the condition stated in Proposition 3-(i).

Next, we examine the derivative of the profit of Firm A, when both firms offer the service, with respect to the degree of network effects of Firm B.

$$\begin{aligned}\frac{\partial}{\partial\alpha_B}\pi_{SS}^A &= \frac{\partial}{\partial\alpha_B}\frac{M\left(3t-M(\alpha_A+2\alpha_B)+s_A-s_B-m_A+m_B\right)^2}{9\left(2t-M(\alpha_A+\alpha_B)\right)^2}\\ &= -\frac{M^2\left(5t-M(3\alpha_A+2\alpha_B)-s_A+s_B+m_A-m_B\right)\left(3t-M(\alpha_A+2\alpha_B)+s_A-s_B-m_A+m_B\right)}{9\left(2t-M(\alpha_A+\alpha_B)\right)^2}\\ &= -D_{SS}^A\,\frac{M\left(5t-M(3\alpha_A+2\alpha_B)-s_A+s_B+m_A-m_B\right)}{3\left(2t-M(\alpha_A+\alpha_B)\right)}\end{aligned} \tag{A32}$$

The above is negative if and only if $\frac{M\left(5t-M(3\alpha_A+2\alpha_B)-s_A+s_B+m_A-m_B\right)}{3\left(2t-M(\alpha_A+\alpha_B)\right)} > 0$. By Assumption 1-i, $2t > M(\alpha_A+\alpha_B)$. Thus, $\frac{\partial}{\partial\alpha_B}\pi_{SS}^A < 0$ if and only if $s_A - s_B < 5t - M(3\alpha_A + 2\alpha_B) + m_A - m_B$. In addition, due to Assumption 1-(i) we have

$$\left(5t - M(3\alpha_A + 2\alpha_B)\right) - \left(3t - M(2\alpha_A + \alpha_B)\right) = 2t - M(\alpha_A + \alpha_B) > 0$$

And due to Assumption 1-(ii) we have $s_A - s_B < 3t - M(2\alpha_A + \alpha_B) + m_A - m_B$, which leads to $s_A - s_B < 5t - M(3\alpha_A + 2\alpha_B) + m_A - m_B$. Therefore, $\frac{\partial}{\partial\alpha_B}\pi_{SS}^A$ is always negative.

(ii) Suppose that in equilibrium Firm A offers the service and Firm B does not. Then, the derivative of Firm A's profit with respect to α_A is

$$\begin{aligned}\frac{\partial}{\partial\alpha_A}\pi_{SN}^A &= \frac{\partial}{\partial\alpha_A}\frac{M\left(3t-c-\alpha_A M+s_A-m_A+m_B\right)^2}{9\left(2t-\alpha_A M\right)}\\ &= \frac{M^2\left(-t+\alpha_A M-c+s_A-m_A+m_B\right)\left(3t-\alpha_A M-c+s_A-m_A+m_B\right)}{9\left(2t-\alpha_A M\right)^2}\\ &= D_{SN}^A\left(\frac{M\left(-t+\alpha_A M-c+s_A-m_A+m_B\right)}{3\left(2t-\alpha_A M\right)}\right)\end{aligned} \tag{A33}$$

Given our assumption that both firms have positive product demands, which also requires $2t > \alpha_A M$, we see that $\frac{\partial}{\partial\alpha_A}\pi_{SN}^A$ is positive if and only if $s_A > t - \alpha_A M + c + m_A - m_B$.

Next we examine the derivative of the profit of Firm B:

$$\begin{aligned}\frac{\partial}{\partial\alpha_A}\pi_{SN}^B &= \frac{\partial}{\partial\alpha_B}\frac{M\left(3t+c-2\alpha_A M-s_A+m_A-m_B\right)^2}{9\left(2t-\alpha_A M\right)}\\ &= \frac{M^2\left(-5t+2\alpha_A M+c-s_A+m_A-m_B\right)\left(3t+c-2\alpha_A M-s_A+m_A-m_B\right)}{9\left(2t-\alpha_A M\right)^2}\\ &= D_{SN}^B\left(\frac{M\left(-5t+2\alpha_A M+c-s_A+m_A-m_B\right)}{3\left(2t-\alpha_A M\right)}\right)\end{aligned} \tag{A34}$$

We see that $\frac{\partial}{\partial\alpha_A}\pi_{SN}^B$ is negative if and only if $\frac{M\left(-5t+2\alpha_A M+c-s_A+m_A-m_B\right)}{3\left(2t-\alpha_A M\right)} < 0$. Given that $2t > \alpha_A M$, we find that Firm B's profit is decreasing in α_A if and only if $s_A > -5t + 2\alpha_A M + c + m_A - m_B$. Furthermore,

$$\left(-5t + 2\alpha_A M + c + m_A - m_B\right) - \bar{s}_A = -\left(2\left(2t - \alpha_A M\right) + \frac{\left(3t-m_i+m_j\right)\sqrt{2\left(2t-\alpha_i M\right)}}{2\sqrt{t}}\right) < 0$$

Thus, $\bar{s}_A > -5t + 2\alpha_A M + c + m_A - m_B$. We conclude that when Firm A offers the service in equilibrium (which implies $s_A > \bar{s}_A$ according to Proposition 1), it must be that $s_A > -5t + 2\alpha_A M + c + m_A - m_B$, and thus $\frac{\partial}{\partial \alpha_A}\pi_{SN}^B < 0$.

(iii) We examine the derivative of the profit of Firm A, when both firms offer the service, with respect to the common degree of network effects:

$$\frac{\partial}{\partial \alpha}\pi_{SS}^A = \frac{\partial}{\partial \alpha}\frac{M\left(3(t-\alpha M)+(s_A-s_B)-m_A+m_B\right)^2}{18(t-\alpha M)} = \frac{M^2}{18}\left(\frac{(s_A-s_B-m_A+m_B)^2}{(t-\alpha M)^2}-9\right) \tag{A35}$$

In equilibrium we have $D_{SS}^A = \frac{1}{6}M\left(3+\frac{(s_A-s_B-m_A+m_B)}{t-\alpha M}\right)$ and $D_{SS}^B = \frac{1}{6}M\left(3+\frac{(s_B-s_A+m_A-m_B)}{t-\alpha M}\right)$. Under our assumption that Firm B has positive demand $\left(D_{SS}^B > 0\right)$, it must be that $\frac{s_A - s_B - m_A + m_B}{t-\alpha M} < 3$. Thus,

$$\frac{\partial}{\partial \alpha}\pi_{SS}^A = \frac{M^2}{18}\left(\frac{(s_A-s_B-m_A+m_B)^2}{(t-\alpha M)^2}-9\right) < 0 \tag{A36}$$

Similarly, $\frac{\partial}{\partial \alpha}\pi_{SS}^B$ is negative when both firms have positive product demand.

Proof of Proposition 4

(i) We examine the derivative of Firm A's profit, when both offer the service, with respect to M.

$$\begin{aligned}\frac{\partial}{\partial M}\pi_{SS}^A &= \frac{\partial}{\partial M}\frac{M\left(3t-M(\alpha_A+2\alpha_B)+s_A-s_B-m_A+m_B\right)^2}{18\left(t-M(\alpha_A+\alpha_B)\right)} \\ &= D_{SS}^A\left(\frac{2}{3}\left(\frac{t\left(s_A-s_B-m_A+m_B+3t-M(\alpha_A+2\alpha_B)\right)}{M\left(2t-M(\alpha_A+\alpha_B)\right)}-\alpha_A-2\alpha_B\right)\right)\end{aligned} \tag{A37}$$

Above is negative if and only if $\frac{t\left(s_A-s_B-m_A+m_B+3t-M(\alpha_A+2\alpha_B)\right)}{M\left(2t-M(\alpha_A+\alpha_B)\right)}-\alpha_A-2\alpha_B$ is negative, which is equivalent to

$$s_A - s_B < 3M\left(\alpha_A+2\alpha_B\right)-3t-\frac{M^2(\alpha_A+\alpha_B)(\alpha_A+2\alpha_B)}{t}+m_A-m_B \tag{A38}$$

The RHS of A38 can be either negative or positive.

(ii) Suppose only Firm A offers the service.

$$\begin{aligned}\frac{\partial}{\partial M}\pi_{SN}^A &= \frac{\partial}{\partial M}\frac{M\left(3t-\alpha_A M+s_A-c-m_A+m_B\right)^2}{9(2t-\alpha_A M)} \\ &= D_{SN}^A\left(\frac{2\left(\alpha_A^2M^2-t\left(c-3t+3\alpha_A M-s_A+m_A-m_B\right)\right)}{3M(2t-\alpha_A M)}\right)\end{aligned} \tag{A39}$$

The above is positive if and only if $\frac{2\left(\alpha_A^2M^2-t\left(c-3t+3\alpha_A M-s_A+m_A-m_B\right)\right)}{3M(2t-\alpha_A M)}$ is positive, which, given the assumption that $t > \alpha_A M$, is equivalent to

$$s_A > c-3t+3\alpha_A M-\frac{\alpha_A^2M^2}{t}+m_A-m_B \tag{A40}$$

Proof of Proposition 5

We start by deriving consumer surplus under each of the four possible market configurations (SS, NN, NS, and SN). Define x_{indif} as the location of the consumer indifferent between buying the product from Firm *A* and buying from Firm *B*. Then, when both firms offer the service in equilibrium

$$x_{indif} = \frac{3t - M(\alpha_A + 2\alpha_B) + s_A - s_B - m_A + m_B}{3(2t - M(\alpha_A + \alpha_B))} \tag{A41}$$

When only Firm *i* sells the service, in equilibrium

$$x_{indif} = \frac{3t - c - \alpha_i M + s_i - m_i + m_j}{6t - 3\alpha_i M} \tag{A42}$$

Consumer surplus when Firm *A* sells the service and Firm *B* does not is given by

$$\begin{aligned} CS_A &= M\int_0^{x_{indif}} \left(V - tx - p_{SN}^A + s_A + \alpha_A M x_{indif}\right)dx + N\int_{x_{indif}}^1 \left(V - t(1-x) - p_{SN}^B\right)dx \\ &= M\int_0^{x_{indif}} \left(V - tx - \left(\tfrac{3t+2c-\alpha_A M + s_A - m_A + m_B}{3}\right) + s_A + \alpha M x_{indif}\right)dx + \\ &\quad M\int_{x_{indif}}^1 \left(V - t(1-x) - \left(\tfrac{3t+c-2\alpha_A M - s_A + m_A - m_B}{3}\right)\right)dx \\ &= MV - \tfrac{M(5t - 3\alpha_A M - 2(s_A - c - m_A - m_B))}{4} \\ &\quad + \tfrac{M(\alpha_A M + 2s_A - 2c - 2m_A - 2m_B)(2t(s_A - c + m_A - m_B) + \alpha_A M(7t - 3\alpha_A M))}{36(2t - \alpha_A M)^2} \end{aligned}$$

Similarly, consumer surplus when only Firm *B* sells the service is given by:

$$\begin{aligned} CS_B &= MV - \frac{M(5t - 3\alpha_B M - 2(s_B - c - m_A - m_B))}{4} \\ &\quad + \frac{M(\alpha_B M + 2s_B - 2c - 2m_A - 2m_B)(2t(s_B - c - m_A + m_B) + \alpha_B M(7t - 3\alpha_B M))}{36(2t - \alpha_B M)^2} \end{aligned}$$

Consumer surplus when both firms offer the service is given by

$$\begin{aligned} CS_{SS} &= M\int_0^{x_{indif}} \left(V - tx - \left(\tfrac{3t - M(\alpha_A + 2\alpha_B) + s_A - s_B + 3c - m_A + m_B}{3}\right) + s_A + \alpha_A M x_{indif}\right)dx + \\ &\quad M\int_{x_{indif}}^1 \left(V - t(1-x) - \left(\tfrac{3t - M(2\alpha_A + \alpha_B) + s_B - s_A + 3c + m_A - m_B}{3}\right) + s_B + \alpha_B M(1 - x_{indif})\right)dx \\ &= MV + \tfrac{M(6(2s_A + s_B) - 18c - 25t + 6M(3\alpha_A + 2\alpha_B) - 12m_A - 6m_B)}{18} + \tfrac{2M(2t - 3\alpha_A M)(t - \alpha_A M - s_A + s_B + m_A - m_B)}{18(2t - M(\alpha_A + \alpha_B))} + \\ &\quad \tfrac{2Mt(t - \alpha_A M - s_A + s_B + m_A - m_B)^2}{18(2t - m(\alpha_A + \alpha_B))^2} \end{aligned}$$

Finally, consumer surplus when neither firm offers the service is given by:

$$
\begin{aligned}
CS_{NN} &= M\int_0^{\frac{3t-m_A+m_B}{6t}} \left(V - tx - \tfrac{3t+2m_A+m_B}{3}\right)dx + M\int_{\frac{3t-m_A+m_B}{6t}}^{1} \left(V - t(1-x) - \tfrac{3t+m_A+2m_B}{3}\right)dx \\
&= MV + \frac{M\left((m_A - m_B)^2 - 18t(m_A + m_B) - 45t^2\right)}{36t}
\end{aligned}
$$

We denote the social welfare when both firms offer service, $\pi^A_{SS} + \pi^B_{SS} + CS_{SS}$, by SW_{SS}, the social welfare when neither firm offers service $\pi^A_{NN} + \pi^B_{NN} + CS_{NN}$, by SW_{NN}, , and the social welfare when only Firm *i* offers service by SW_i. The profit expressions are given in Table 3, and were derived in Appendix A.

$F^i(s_i, s_j)$ is defined as the difference between social welfare when both firms offer service to social welfare when only Firm *i* offers service, specifically:

$$
\begin{aligned}
&F^i(s_i, s_j) = SW_{SS} - SW_i = \\
&\tfrac{M}{9}\Bigg(7s_i + s_j + M(5\alpha_i + \alpha_j) - 8c - 7(m_i - m_i) + \tfrac{t\left(t-\alpha_i M+s_j-s_i+m_i-m_j\right)^2}{\left(2t-M(\alpha_i+\alpha_j)\right)^2} + \\
&\tfrac{\left(4t-5\alpha_i M+2(s_j-s_i+m_i-m_j)\right)\left(t-\alpha_i M-s_i+s_j+m_i-m_j\right)}{2t-M(\alpha_i+\alpha_j)} - \\
&\tfrac{2\left(2t+s_i-c-m_i-m_j\right)\left(t+s_i+c-m_i+m_j\right)}{2t-\alpha_i M} - \tfrac{t\left(c+m_i-m_j-s_i-t\right)^2}{(2t-\alpha_i M)^2}\Bigg)
\end{aligned}
\tag{A43}
$$

Thus, when $F^i(s_i, s_j) < 0$, social welfare when only Firm *i* offers the service exceeds social welfare when both firms offer the service.

Setting $m_i = m_j$, $F^i(s_i, s_j)$ becomes

$$
\begin{aligned}
&F^i(s_i, s_j) = \\
&\tfrac{M}{9}\Bigg(7s_i + s_j + M(5\alpha_i + \alpha_j) - 8c + \tfrac{t\left(t-\alpha_j M+s_j-s_i\right)^2}{\left(2t-M(\alpha_i+\alpha_j)\right)^2} + \tfrac{\left(4t-5\alpha_i M+2(s_j-s_i)\right)\left(t-\alpha_i M-s_i+s_j\right)}{2t-M(\alpha_i+\alpha_j)} - \\
&\tfrac{2(2t+s_i-c)(t+s_i+c)}{2t-\alpha_i M} - \tfrac{t(c-s_i-t)^2}{(2t-\alpha_i M)^2}\Bigg)
\end{aligned}
$$

When $m_A = m_B$, given the conditions on s_i specified in Assumption 1, we can show that $SW_A > SW_{NN}$ iff $s_A > \frac{2c-\alpha_A N}{2}$. Similarly, $SW_B > SW_{NN}$ iff $s_B > \frac{2c-\alpha_B N}{2}$. In addition, it is easy to show that $\frac{2c-\alpha_i N}{2} < \bar{s}_i$. Thus, as long as in equilibrium at least one firm offers the service (i.e., at least one s_i is larger than $\bar{s}_i$), we know that NN is not socially optimal. As long as $s_A > \frac{2c-\alpha_A N}{2}$ or $s_B > \frac{2c-\alpha_B N}{2}$ (or both), social welfare when one firm offers service exceeds social welfare when neither offers, and thus social welfare is maximized when both offer service if and only if $F^A(s_A, s_B) > 0$and $F^B(s_A, s_B) > 0$.

Finally, when $s_A < \frac{2c-\alpha_A N}{2}$ and $s_B < \frac{2c-\alpha_B N}{2}$, social welfare when neither firm offers service is larger than social welfare when only Firm *A* or only Firm *B* offers the service. In addition, when $s_A < \frac{2c-\alpha_A N}{2}$, $s_B < \frac{2c-\alpha_B N}{2}$, and $c > \frac{\alpha_i N}{2}$, we find that $SW_{SS} < SW_{NN}$. Finally, when $s_A < \frac{2c-\alpha_A N}{2}$ and $s_B < \frac{2c-\alpha_B N}{2}$, in equilibrium, neither firm offers service (as $\frac{2c-\alpha_i N}{2} < \bar{s}_i$). Thus the equilibrium is NN, which is also socially optimal. The rest is trivial based on the results from Proposition 1.

Proof of Proposition 6

In the case in which firms choose the direct service quality (s_i) endogenously, to ensure that the second-order conditions are met, the market is covered, and the two firms have positive demands, the following parameter assumptions are needed.

Assumption 2.

(i) $t > \alpha_i M$ ($i = A$ and B)

(ii) $c_i > \frac{M}{18(t-\alpha_i M)}$ ($i = A$ and B)

(iii) $c_i > \frac{M}{3(3t-2\alpha_i M+c)}$ ($i = A$ and B)

(iv) $c < 3t - \alpha_i M$ ($i = A$ and B)

(v) $c_i < \frac{M}{9(t-\alpha_o M)}$ ($i = A$ and B)

(i) In Case SN,

$$\frac{\partial}{\partial \alpha_A} s_A = \frac{M^2\left(M-9c_A(c-t)\right)}{\left(M-9c_A(2t-\alpha_A M)\right)^2}$$

This is positive if and only if (i) $c > t$ and $c_A < \frac{M}{9(c-t)}$ or (ii) $c < t$ and $c_A > \frac{M}{9(c-t)}$. In the latter case, $\frac{M}{9(c-t)} < 0$ and thus, when $c < t$, we have $\frac{\partial}{\partial \alpha_A} s_A > 0$ for all positive c_A.

(ii) In Case SS,

$$\frac{\partial}{\partial \alpha_A} s_A = \frac{c_B M^2\left(c_B M - c_A\left(2M - 9c_B(t-\alpha_A M)\right)\right)}{\left(M(c_A+c_B) - 9c_A c_B\left(2t - M(\alpha_A+\alpha_B)\right)\right)^2}$$

By Assumption 2-(v), $2M - 9c_B(t - \alpha_B M) = M + \left(M - 9c_B(t - \alpha_B M)\right)$. Therefore, $\frac{\partial}{\partial \alpha_A} s_A > 0$ if and only if $c_A < \frac{c_B M}{2M - 9c_B(t-\alpha_B M)}$.

$$\frac{\partial}{\partial \alpha_B} s_A = \frac{c_B M^2\left(2c_B M - c_A\left(M + 9c_B(t-\alpha_A M)\right)\right)}{\left(M(c_A+c_B) - 9c_A c_B\left(2t - M(\alpha_A+\alpha_B)\right)\right)^2}$$

This is positive if and only if $c_A < \frac{2c_B M}{M+9c_B(t-\alpha_A M)}$.

(iii) When $\alpha_A = \alpha_B = \alpha$, the optimal direct value is $s_A = \frac{M\left(M-9c_B(t-\alpha M)\right)}{M(c_A+c_B)-18c_A c_B(t-\alpha M)}$. Then

$$\frac{\partial}{\partial \alpha} s_A = \frac{3c_B M^2(c_B - c_A)}{\left(M(c_A+c_B) - 18c_A c_B(t-\alpha M)\right)^2}$$

which is positive if any only if $c_A < c_B$.

Proof of Proposition 7

(i) In Case SN,

$$\frac{\partial}{\partial \alpha_A}\pi_{SN}^A = \frac{\partial}{\partial \alpha_A}\left(\frac{3c_A M(3t-\alpha_A M-c)^2}{9c_A(2t-\alpha_A M)-M}\right) = \frac{c_A M^2(3t-\alpha_A M)(2M-9c_A(t-\alpha_A M+c))}{(9c_A(2t-\alpha_A M)-M)^2}$$
$$= D_{SN}^A \frac{M(2M-2c_A(t-\alpha_A M+c))}{3(9c_A(2t-\alpha_A M)-M)}$$

We can show that D_{SN}^A is positive by Assumption 2 and $9c_A(2t-\alpha_A M)-M>0$ by Assumption 2-(ii). Thus, $\frac{\partial}{\partial \alpha_A}\pi_{SN}^A > 0$ if and only if $2M - 9c_A(t-\alpha_A M+c)>0$, which is equivalent to $\alpha_A > \frac{c+t}{M} - \frac{2}{9c_A}$.

(ii)

$$\frac{\partial}{\partial \alpha_A}\pi^A = -\frac{2c_A^2 c_B M^2(9c_A(2t-M(\alpha_A+\alpha_B))-M)(M-9c_B+c_B M(3\alpha_A+6\alpha_B))^2}{9(M(c_A+c_B)-9c_A c_B(2t-M(\alpha_A+\alpha_B)))^2}$$
$$-\frac{3c_A M^2(M-9c_B t+c_B M(3\alpha_A+6\alpha_B))(M(3c_A+2c_B)-9c_A c_B(7t-M(3\alpha_A+4\alpha_B)))}{9(M(c_A+c_B)-9c_A c_B(2t-M(\alpha_A+\alpha_B)))^2}$$

The first term of $\frac{\partial}{\partial \alpha_A}\pi^A$ is negative because $9c_A(2t-M(\alpha_A+\alpha_B))-M>0$ and $M(c_A+c_B)-9c_A c_B(2t-M(\alpha_A+\alpha_B))>0$ by Assumption 2-(ii) and (v). Also by Assumption 2-(iv), $M-9c_B+c_B M(3\alpha_A+6\alpha_B)>0$. Thus, the second term is negative if

$$M(3c_A+2c_B)-9c_A c_B(7t-M(3\alpha_A+4\alpha_B))>0$$

which is equivalent to $\alpha_A > \frac{1}{27}\left(\frac{36t}{M} - 36\alpha_B - \left(\frac{2}{c_A}+\frac{3}{c_B}\right)\right)$.

(iii) When $\alpha_A = \alpha_B = \alpha$ and both firms offer the service,

$$\pi^A = \frac{c_A M(18c_A(t-\alpha M)-M)(M-9c_B(t-\alpha M))^2}{9(M(c_A+c_B)-18c_A c_B(t-\alpha M))^2}$$
$$\frac{\partial}{\partial \alpha}\pi^A = D^A \frac{2M\left(-c_A^2\left(2M^2(M-9c_B(t-\alpha M))+(18c_B(t-\alpha M)-M)+27c_A c_B^2 M(t-\alpha M)-2c_B^2 M^2\right)\right)}{9(M(c_A+c_B)-18c_A c_B(t-\alpha M))^2}$$

Therefore $\frac{\partial}{\partial \alpha}\pi^A > 0$ if and only if $-c_A^2\left(2M^2(M-9c_B(t-\alpha M))+(18c_B(t-\alpha M)-M)\right)+27c_A c_B^2 M(t-\alpha M)-2c_B^2 M^2>0$. As the coefficient of c_A^2 is negative by Assumption 2-(ii) and (v), $\frac{\partial}{\partial \alpha}\pi^A > 0$ if and only if

$$\frac{2c_B M}{27c_B(t-\alpha M)+\sqrt{81c_B^2(t-\alpha M)^2+36c_B M(t-\alpha M)-4M^2}} < c_A < \frac{2c_B M}{27c_B(t-\alpha M)-\sqrt{81c_B^2(t-\alpha M)^2+36c_B M(t-\alpha M)-4M^2}}$$

However, when $c_A = \frac{M}{18(t-\alpha M)}$ (the lower bound of c_A given by Assumption 2-ii), $\frac{\partial}{\partial \alpha}\pi^A = D_A \frac{M^3(M-9c_B(t-\alpha M))}{1458(M(c_A+c_B)-18c_A c_B(t-\alpha M))^2} > 0$ by Assumption 2-(iv). Also, if $c_A = c_B$ then $\frac{\partial}{\partial \alpha}\pi^A = D_A \frac{-4c_B^2(M-9c_B(t-\alpha M))^2}{9(M(c_A+c_B)-18c_A c_B(t-\alpha M))^2} < 0$.

Therefore, $\frac{\partial}{\partial \alpha}\pi^A > 0$ if and only if $c_A < \frac{2c_B M}{27c_B(t-\alpha M)-\sqrt{81c_B^2(t-\alpha M)^2+36c_B M(t-\alpha N)-4M^2}} < c_B$.

Proof of Proposition 8

(i) For example, in Case SN,

$$\frac{\partial}{\partial c_A} s_{SN}^A = \frac{\partial}{\partial c_A} \frac{M(3t-\alpha_A M-c)}{9c_A(2t-\alpha_A M)-M} = -\frac{9M(2t-\alpha_A M)(3t-\alpha_A M-c)}{\left(9c_A(2t-\alpha_A M)-M\right)^2} = -D_{SN}^A \frac{3(2t-\alpha_A M)}{c_A\left(9c_A(2t-\alpha_A M)-M\right)} < 0$$

$$\frac{\partial}{\partial c_A} \pi_{SN}^A = \frac{\partial}{\partial c_A} \left(\frac{c_A M(3t-\alpha_A M-c)^2}{9c_A(2t-\alpha_A M)-M} \right) = -\frac{M^2(3t-\alpha_A M-c)^2}{\left(9c_A(2t-\alpha_A M)-M\right)^2} < 0$$

$$\frac{\partial}{\partial c_A} \pi_{SN}^B = \frac{\partial}{\partial c_A} \left(\frac{M(2t-\alpha_A M)\left(3c_A(3t-2\alpha_A M+c)-M\right)^2}{\left(9c_A(2t-\alpha_A M)-M\right)^2} \right)$$

$$= \frac{6M^2(2t-\alpha_A M)(3t-\alpha_A M-c)\left(3c_A(3t-2\alpha_A M+c)-M\right)}{\left(9c_A(2t-\alpha_A M)-M\right)^3} = D_{SN}^B \left(\frac{6M(2t-\alpha_A M)(3t-\alpha_A M-c)}{\left(9c_A(2t-\alpha_A M)-M\right)^2} \right)$$

In $\frac{\partial}{\partial c_A} \pi_{SN}^B$, $3t - \alpha_A M - c > 0$ by Assumption 2-(iv). Thus, $\frac{\partial}{\partial c_A} \pi_{SN}^B > 0$.

(ii) When both firms offer the service,

$$\frac{\partial}{\partial c_A} s^A = \frac{\partial}{\partial c_A} \frac{M\left(M-9c_B t+c_B M(3\alpha_A+6\alpha_B)\right)}{3M(c_A+c_B)-27c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)}$$

$$= \frac{M^2\left(M-9c_B t+c_B M(3\alpha_A+6\alpha_B)\right)\left(9c_B\left(2t-M(\alpha_A+\alpha_B)\right)-M\right)}{3\left(M(c_A+c_B)-9c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)\right)^2}$$

By Assumption 2-(ii) and (v), the numerator of $\frac{\partial}{\partial c_A} s^A$ is positive. Thus, $\frac{\partial}{\partial c_A} s^A > 0$.

$$\frac{\partial}{\partial c_A} \pi^A = \frac{\partial}{\partial c_A} \left(\frac{c_A M\left(9c_A\left(2t-M(\alpha_A+\alpha_B)\right)-M\right)\left(M-9c_B t+c_B M(3\alpha_A+6\alpha_B)\right)^2}{9\left(M(c_A+c_B)-9c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)\right)^2} \right)$$

$$= \frac{M^2\left(M-9c_B t+c_B M(3\alpha_A+6\alpha_B)\right)^2\left(c_A M+c_B\left(9c_A\left(2t-M(\alpha_A+\alpha_B)\right)-M\right)\right)}{9\left(M(c_A+c_B)-9c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)\right)^3}$$

$c_A M + c_B\left(9c_A\left(2t - M(\alpha_A + \alpha_B)\right) - M\right)$ is positive by Assumption 2-(ii). Thus, $\frac{\partial}{\partial c_A} \pi^A > 0$.

$$\frac{\partial}{\partial c_B} s^A = -\frac{M\left(M-9c_A t+c_A M(6\alpha_A+3\alpha_B)\right)}{3\left(M(c_A+c_B)-9c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)\right)^2}$$

The numerator of $\frac{\partial}{\partial c_B} s^A$ is positive by Assumption 2-(v). Thus, $\frac{\partial}{\partial c_B} s^A < 0$.

$$\frac{\partial}{\partial c_B} \pi^A = -\frac{2c_A M^2\left(M-9c_A t+c_A M(6\alpha_A+3\alpha_B)\right)\left(M-9c_B t+c_B M(3\alpha_A+6\alpha_B)\right)\left(9c_A\left(2t-M(\alpha_A+\alpha_B)\right)-M\right)}{9\left(M(c_A+c_B)-9c_A c_B\left(2t-M(\alpha_A+\alpha_B)\right)\right)^3}$$

Similarly, by Assumption 2-(ii) and (v), the numerator and denominator are positive. Thus, $\frac{\partial}{\partial c_B} \pi^A < 0$.

Reference

Li, X., and Chen. Y. 2012. "Corporate IT Standardization: Product Compatibility, Exclusive Purchase Commitment, and Competition Effects," *Information Systems Research* (23:4), pp. 1158-1174.