MIS Quarterly

RESEARCH ARTICLE

An Analysis of Pricing Models in the Electronic Book Market

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Appendix

Proof of Proposition 1

We solve p_D^* from $\frac{\partial \pi_R}{\partial p_D} = 0$. Then we substitute it into $\frac{\partial \pi_R}{\partial p_E} = 0$ and solve for p_E^*. We obtain $p_E^* = w_E$ or $p_E^* = w_E \pm \sqrt{A_1}$ where A_1 is a function of the parameters. We find $\pi_R\big|_{p_E^*=w_E} - \pi_R\big|_{p_E^*=w_E\pm\sqrt{A_1}} = \frac{A_1^2}{32m^2t} > 0$. Therefore, we have $\pi_R\big|_{p_E^*=w_E} > \pi_R\big|_{p_E^*=w_E\pm\sqrt{A_1}}$. Next, we find $\pi_P\big|_{p_E^*=w_E} - \pi_P\big|_{p_E^*=0} = \frac{w_E^2 B_1}{32m^2t}$ where B_1 is a function of the parameters. Remember that the e-book market size s_E must be greater than zero at $p_E^* = 0$ in order to make $p_E^* = 0$ economically sensible. Therefore, we have $s_E\big|_{p_E^*=0} = \frac{C_1}{16mt} > 0$. We find $B_1 - C_1 = w_E^2 > 0$. As $C_1 > 0$, we find $B_1 > 0$. Since $B_1 > 0$, we have $\pi_P\big|_{p_E^*=w_E} > \pi_P\big|_{p_E^*=0}$. To verify the second order conditions, we derive the 2 × 2 Hessian matrix. We evaluate matrix H at optimal point (p_E^*, p_D^*). We find $H_{11} = \frac{\partial^2 \pi_R}{\partial p_E^2} = \frac{-1}{8}\frac{D_1}{m^2t}$ and $|H| = \frac{E_1}{8t^2m^2}$ where D_1 and $E_1 = D_1 - 8(b - w_E)^2$. Therefore, we find $D_1 > E_1$. So in order to show $H_{11} < 0$ and $|H| > 0$, we only need to show $E_1 > 0$. We find $s_E\big|_{p_E^*=w_E} = \frac{E_1}{8mt}$. We know $s_E\big|_{p_E^*=w_E}$ must be greater than zero. Hence we have $E_1 > 0$. So we verify that the optimal choice of e-book retail price p_E^* and e-reader price p_D^* in Proposition 1.

Proof of Proposition 2

We let $\hat{\pi}_P$ denote the publisher's total profit at the retailer's optimal choice of prices (p_E^*, p_D^*). First we wil prove that $0 < w_E^* < \frac{b + c_A}{2}$. We define $f_2(w_E) = \frac{\partial \hat{\pi}_P}{\partial w_E}$. Notice that f_2 is a cubic function of w_E and we find that $\frac{\partial f_2}{\partial w_E^3} = -\frac{3}{tm^2} < 0$. The potential optimal points are $f_2(w_E^*) = 0$, $w_E^* = 0$, or $w_E^* = b$. We find that $w_E^* = 0$ can be removed from the candidate set as it is not difficult to show $\hat{\pi}_P\big|_{w_E^*=0}$ is less than the profit of not selling any e-book. Second, $w_E^* = b$ can also be removed as it violates consumer's IR constraint. Therefore, we focus on $f_2(w_E^*) = 0$. Next, we show $f_2\big|_{w_E=0} > 0$. We derive $f_2\big|_{w_E^*=0} = \frac{A_2}{8tm^2}$. It is not difficult to show

$\min A_2 = 2\left(\left(w_F - c_A - c_P\right)\left(b - p_F\right) + c_A b\right)b > 0$. Therefore, we have $f_2\big|_{w_E=0} > 0$. Next we find $f_2\big|_{w_E=(b+c_A)/2} = -\frac{(b-c_A)B_2}{32tm^2}$ and $\min_{c_A} B_2 = 4\left(p_F - w_F + c_P\right)\left(b - p_F\right) > 0$. Therefore, we have $f_2\big|_{w_E=(b+c_A)/2} < 0$. Meanwhile, we find that $\frac{\partial^2 f_2}{\partial w_E^2}\big|_{w_E=(b+c_A)/2} = \frac{3}{4}\frac{b-c_A}{tm^2} > 0$. Combining $f_2\big|_{w_E=0} > 0$, $f_2\big|_{w_E=(b+c_A)/2} < 0$, $\frac{\partial^2 f_2}{\partial w_E^2}\big|_{w_E=(b+c_A)/2} = \frac{3}{4}\frac{b-c_A}{tm^2} > 0$, and $\frac{\partial^3 f_2}{\partial w_E^3} < 0$, we conclude that there is only one w_E^* in $\left(0, \frac{b+c_A}{2}\right)$, satisfying $f_2(w_E^*) = 0$. Because $\frac{\partial^2 \hat{\pi}_P}{\partial w_E^2}\big|_{w_E=w_E^*} < 0$, $\frac{\partial w_E^*}{\partial c_P}$ has the same sign as $\frac{\partial^2 \hat{\pi}_P}{\partial w_E \partial c_P}\big|_{w_E=w_E^*}$. We derive $\frac{\partial^2 \hat{\pi}_P}{\partial w_E \partial c_P}\big|_{w_E=w_E^*} = -\frac{(b-p_F)(b-w_E^*)}{4m^2 t} < 0$. Therefore, we find $\frac{\partial w_E^*}{\partial c_P} < 0$. Similarly, $\frac{\partial w_E^*}{\partial c_F}$ has the same sign as $\frac{\partial^2 \hat{\pi}_P}{\partial w_E \partial c_F}\big|_{w_E=w_E^*}$. We then derive $\frac{\partial^2 \hat{\pi}_P}{\partial w_E \partial c_F}\big|_{w_E=w_E^*} = \frac{(b-p_F)(b-2w_E^*+c_A)}{4m^2 t}$. Given $0 < w_E^* < \frac{b+c_A}{2}$, we have $\frac{\partial^2 \hat{\pi}_P}{\partial w_E \partial c_F}\big|_{w_E=w_E^*} > 0$. Hence, we find $\frac{\partial w_E^*}{\partial c_F} > 0$.

Proof of Proposition 3

Note that π_R is a quadratic function for p_D and $\partial^2\pi_R/\partial p_D^2 < 0$. Therefore, we solve $\partial\pi_R/\partial p_D = 0$ and then we find p_D^* in Proposition 3.

Proof of Proposition 4

Proof of this proposition follows the procedure similar to Proposition 2. We let $\hat{\pi}_P$ denote the publisher's total profit at the retailer's optimal choice of e-reader price p_D^*. We define $f_4(p_E) = \frac{\partial \hat{\pi}_P}{\partial p_E}$. Notice that f_4 is a cubic function of p_E when $r \neq \frac{1}{2}$ (in the case of $r \neq \frac{1}{2}$, the proof will be straightforward). Next, we show $f_4\big|_{p_E=0} > 0$. We derive $f_4\big|_{p_E=0}$ and follow the similar steps as the proof of Proposition 2. We find that $\min f_4\big|_{p_E=0} = \frac{2b(1-r)\cdot\left(\left(w_F - c_P - c_A\right)\left(b-p_F\right) + bc_A\right)}{8tm^2} > 0$. So we have $f_4\big|_{p_E=0} > 0$.

Next we find $f_4\big|_{p_E=\bar{p}_E} = \frac{\left(c_A - 2c_A r - b + rb\right)A_2}{32m^2(1-r)^2 t}$ and $\min_{c_A} A_4 = 4(1-r)\left(p_F - p_F r + c_P - w_F\right)\left(b - p_F\right)$. It is not difficult to show $c_A - 2c_A r - b + rb < 0$. Therefore, $f_4\big|_{p_E=\bar{p}_E} < 0$ when $r < \frac{p_F - w_F + c_P}{p_F}$. Meanwhile, we find that $\frac{\partial^2 f_4}{\partial p_E^2}\big|_{p_E=\bar{p}_E} = -\frac{3}{4}\frac{c_A - 2rc_A - b + rb}{m^2 t} > 0$. Combining $f_4\big|_{p_E=0} > 0$, $f_4\big|_{p_E=\bar{p}_E} < 0$, and $\frac{\partial^2 f_4}{\partial p_E^2}\big|_{p_E=\bar{p}_E} > 0$ and the fact that f_4 is a cubic function, we conclude that there exists only one p_E^* in $(0, \bar{p}_E)$ satisfying $f_4(p_E^*) = 0$. Therefore, we have $p_E^* < \bar{p}_E$ when $r < \frac{p_F - w_F + c_P}{p_F}$.

Proof of Proposition 5

Having shown the result $0 < p_E^* < \bar{p}_E$ in Proposition 4, we will next prove that $\frac{\partial p_E^*}{\partial c_P} < 0$ and $\frac{\partial p_E^*}{\partial c_F} > 0$. $\frac{\partial p_E^*}{\partial c_P} < 0$ Has the same sign as $\frac{\partial^2 \hat{\pi}_P}{\partial p_E \partial c_P}\big|_{p_E=p_E^*}$. We derive $\frac{\partial^2 \hat{\pi}_P}{\partial p_E \partial c_P}\big|_{p_E=p_E^*} = \frac{\left(b-p_F\right)\left(p_E - 2p_E r - b + rb\right)}{4m^2 t}$. It is not difficult to show $p_E - 2p_E r - b + rb < 0$. Hence, we find $\frac{\partial p_E^*}{\partial c_P} < 0$. Similarly, $\frac{\partial p_E^*}{\partial c_F}$ has the same sign as $\frac{\partial^2 \hat{\pi}_P}{\partial p_E \partial c_P}\big|_{p_E=p_E^*}$. We then derive

$\left.\frac{\partial^2 \hat{\pi}_P}{\partial p_E \partial c_F}\right|_{p_E=p_E^*} = \frac{(1-r)(b-p_F)(\bar{p}_E - p_E)}{2m^2 t}$. Given $0 < p_E^* < \bar{p}_E$, we have $\left.\frac{\partial^2 \hat{\pi}_p}{\partial p_E \partial c_F}\right|_{p_E=p_E^*} > 0$. Therefore, we proved $\frac{\partial p_E^*}{\partial c_F} > 0$.

Proof of Proposition 6

In this subsection of proof, we denote the publisher's profit in the agency model's SPNE by $\hat{\pi}_P^A$. We denote the publisher's profit in the wholesale model's SPNE by $\hat{\pi}_P^W$. We align the two first order conditions $\partial\hat{\pi}_P^A/\partial p_E = 0$ and $\partial\hat{\pi}_P^W/\partial w_E = 0$. We eliminate a common term $m(t - c_D) - w_F p_F + c_F b + p_F^2/2 - c_F p_F + w_F b$ and then obtain a new equation $f_8(p_E^{A*}, w_E^{W*}) = 0$. Since $w_E^{W*} = p_E^{W*}$ in the wholesale model's SPNE, in order to prove $p_E^{A*} > p_E^{W*}$, we just need to prove $p_E^{A*} > w_E^{W*}$. We define $w_E^{W*} = kp_E^{A*}$ and substitute it into $f_8(p_E^{A*}, w_E^{W*}) = 0$. This transforms $f_8(p_E^{A*}, w_E^{W*}) = 0$ into $f_8(k) = 0$. We define $f_9(k) = m^2 t \cdot f_8(k)$. Then we just need to prove that there exists one and only one root of $f_9(k)$ in [0, 1]. We will prove it by proving $f_9\big|_{k=0} < 0$, $f_9\big|_{k=1} > 0$, and $\partial f_9 / \partial k > 0$ in $0 < k \le 1$. In the following, for simplicity, we denote p_E^{A*} by p_E and denote w_E^{W*} by w_E.

First we will prove $\partial f_9/\partial k > 0$ in $0 < k \le 1$. We derive $\frac{\partial f_9}{\partial k} = -\frac{1}{4}\frac{p_E \cdot f_{11}(p_E)\cdot f_{10}(p_E)}{(b - 2kp_E + c_A)^2}$ where $f_{10}(p_E) = rb - b - c_A + 2\,p_E - 2p_E r$ and $f_{11}(p_E)$ is a cubic function of p_E with other parameters. It is not difficult to show $f_{10}(p_E) < 0$ when $p_E < \bar{p}_E = (b + c_A/(1-r))/2$. Next we will prove $f_{11}(p_E)$. In Proposition 2.3, we have proved $w_E < \bar{w}_E = (b + c_A)/2$. Since we have $w_E = kp_E$, we have $p_E < \hat{p}_E = (b + c_A)/(2k)$. We derive $f_{12}(p_E) = \frac{\partial^2 f_{11}}{\partial p_E^2} = -48k^3 p_E + 18k^2 c_A + 30k^2 b$ and $f_{13}(p_E) = \frac{\partial f_{11}}{\partial p_E} = -24k^3 p_E^2 + 2(9k^2 c_A + 15k^2 b)p_E - 12kc_A b - 9kb^2 - 3kc_A^2$. We first find $f_{12}\big|_{p_E=\hat{p}_E} = 6k^2(b - c_A) > 0$. Second, when $0 < k \le 1$, we find $f_{12}\big|_{p_E=0} = 6k^2(5b + 3c_A) > 0$. Therefore, $f_{12}(p_E) > 0$. Next, we find $f_{13}\big|_{p_E=\hat{p}_E} = 0$ and $f_{13}\big|_{p_E=0} = -3k(c_A + b)(3b + c_A)$. Together with $f_{12}(p_E) = \partial f_{13}/\partial p_E > 0$, we find $f_{13}(p_E) < 0$. Therefore, in order to show $f_{11}(p_E) > 0$, we just need to prove $f_{11}\big|_{p_E=\hat{p}_E} > 0$. We find $\left.f_{11}\right|_{p_E=\hat{p}_E} = \frac{1}{4}(b - c_A)f_{14}$ and $\min_{c_A} f_{14} = 4(p_F - w_F + c_P)(b - p_F) > 0$. Therefore, we proved $f_{11}(p_E) > 0$. Together with $f_{10}(p_E) < 0$, we proved $\partial f_9/\partial k > 0$ in $0 < k \le 1$.

Next, we will prove $f_9\big|_{k=0} < 0$. We define $f_{15}(p_E) = f_9\big|_{k=0}$ (i.e., we treat $f_9\big|_{k=0}$ as a function of p_E). Notice that $f_{15}(p_E)$ is a cubic function of p_E. We derive $\left.f_{15}\right|_{p_E=0} = -\frac{1}{4}\frac{c_A rb((b - p_F)(w_F - c_P) + c_A p_F)}{c_A + b}$. We find that $f_{15}\big|_{p_E=0} < 0$. Next, we find $\left.f_{15}\right|_{p_E=\bar{p}_E} = \frac{1}{32}\frac{(c_A - 2rc_A - b + rb)f_{18}}{(1-r)^2}$ and $\min_{c_A} f_{18} = -4(1-r)(p_F r - c_P + w_F - p_F)(b - p_F)$. It is not difficult to show when $r < \frac{p_F - w_F + c_P}{p_F}$, we have $f_{15}\big|_{p_E=\bar{p}_E} < 0$. We define $f_{19}(p_E) = \partial f_{15}^2 / \partial p_E^2$. We find that $f_{19}(p_E)$ is a linear function of p_E. We find that $f_{19}\big|_{p_E=0} > 0$ when $r \le \frac{3}{4}$. We also find that when $0 < r < 1$ we have $f_{19}\big|_{p_E=\bar{p}_E} > 0$. Therefore we prove $\partial f_{15}^2 / \partial p_E^2 > 0$ in $p_E \in (0, \bar{p}_E)$ when $r \le \frac{3}{4}$. Combining $f_{15}\big|_{p_E=0} < 0$, $f_{15}\big|_{p_E=\bar{p}_E} < 0$, and $\partial f_{15}^2 / \partial p_E^2 > 0$, we find $f_9\big|_{k=0} = f_{15}(p_E) < 0$ when $r \le \frac{3}{4}$.

Next, we will prove $f_9\big|_{k=1} > 0$. We define $f_{20}(r) = f_9\big|_{k=1}$ as a quadratic equation of r. We derive $f_{20}\big|_{r=0} = 0$ and $\partial^2 f_{15} / \partial r^2 = -(b - w_E)(b - 2w_E)w_E$. In order to show $f_9\big|_{k=1} > 0$, we need $w_E < b/2$ and $f_{20}\big|_{r=1} > 0$. We derive $\left.f_{20}\right|_{r=1} = \frac{f_{21}(w_E)}{4(b - 2w_E + c_A)}$

where $f_{21}(w_E)$ is a cubic function of w_E. We find that $f_{21}\big|_{w_E=c_A} = 2c_A^2(b-c_A)^2 > 0$ and $f_{21}\big|_{w_E=b/2} = \frac{1}{8}b^2c_A(2c_A+b) > 0$. We also find that $\partial^3 f_{21} / \partial w_E^3 > 0$ and $\frac{\partial^2 f_{21}}{\partial w_E^2}\big|_{w_E=b/2} < 0$ when $p_F < b$. Therefore, we find $f_{20}\big|_{r=1} > 0$ when $c_A < w_E < b/2$. Therefore, to make $f_9\big|_{k=1} > 0$, we only need $c_A < w_E < b/2$. Following the same procedure of proving $0 < w_E < (b + c_A)/2$ in Proposition 2, we find that a sufficient condition for $c_A < w_E < b/2$ is $w_F - c_P < p_F/2$ and $c_A \le p_F/4$. Therefore, we prove $p_E^{A*} > p_E^{W*}$ when $r \le \min\left\{\frac{3}{4}, \frac{p_F - w_F + c_P}{p_F}\right\}$, $w_F - c_P < p_F/2$ and $c_A \le p_F/4$. Regarding the e-reader prices, we derive $p_D^{A*} - p_E^{W*} = -\frac{1}{4}\left(p_E^{A*} - p_E^{W*}\right)\left(2b - p_D^{A*} - w_E^{W*}\right) - \frac{1}{2}rp_E^{A*}\left(b - p_E^{A*}\right)$. Since we find that when $p_E^{A*} > p_E^{W*}$, we have $p_D^{A*} < p_D^{W*}$.

Proof of Proposition 7 and Proposition 8

For the wholesale mode, we solve the retailer's constraint optimization problem using the Lagrange method. From the first order condition, we obtain

$p_{D1}^* = \left(t + \bar{c}_D\right)/2 + \left(A \cdot \bar{\theta}\right)/(4m) + \left(\theta_H(b-p_E)^2(1-k)(2-a)\right)/(4m)$ and $p_{D2}^* = \left(t + \bar{c}_D\right)/2 + \left(A \cdot \bar{\theta}\right)/(4m) - \left(\theta_H(b-p_E)^2(1-k)a\right)/(4m)$ where $\bar{\theta} = a\theta_H + (1-a)\theta_L$, $\bar{c}_D = ac_{D1} + (1-a)c_{D2}$, and $A = 2(b - p_F)(p_F - w_F - c_F) + k(b - p_E)^2 - 2(p_E - w_E)(b - p_E) - (b - p_F)^2$. Consider p_{D1}^* and p_{D2}^* as functions of p_E. We plug them back to the retailer's profit function π_R. We define that $g_H(p_E) = \left((p_E - w_E)q_{EH} + \left(p_{D1}^* - c_{D1}\right)\right)s_{EH} + (p_F - w_F - c_F)q_{FH}s_{FH}$, $g_L(p_E) = \left((p_E - w_E)q_{EL} + \left(p_{D2}^* - c_{D2}\right)\right)s_{EL} + (p_F - w_F - c_F)q_{FL}s_{FL}$. The π_R can be expressed by $\pi_R = ag_H + (1-a)g_L$. Following the similar steps of the proof in Proposition 2, it is not difficult to show that (i) $\partial g_H/\partial p_E = 0$ can only be attained at p_{EH}^* where $p_{EH}^* > w_E$, (ii) $\partial g_L/\partial p_E = 0$ can only be attained at p_{EL}^* where $p_{EL}^* > w_E$, (iii) $\partial^2 g_H / \partial p_E^2\big|_{p_E=p_{EH}^*} < 0$ and $\partial^2 g_L / \partial p_E^2\big|_{p_E=p_{EL}^*} < 0$. As π_R is a convex combination of g_H and g_L, we find $p_E^* > w_E$. For the agency model, it is straightforward to show the results presented in Proposition 8 through solving the first order conditions using the Lagrange method.