Appendix

Proof of Proposition 1

We solve \( p^*_D \) from \( \frac{\partial \pi_s}{\partial p_D} = 0 \) and solve for \( p^*_E \). We obtain \( p^*_E = w_E \) or \( p^*_E = w_E \pm \sqrt{A_1} \) where \( A_1 \) is a function of the parameters. We find \( \pi_s \bigg|_{p^*_E = w_E} \) \( > 0 \) and \( \pi_s \bigg|_{p^*_E = w_E \pm \sqrt{A_1}} \) \( > 0 \). Next, we find \( \pi_s \bigg|_{p^*_E \neq w_E} \). Notice that \( B_1 \) is a function of the parameters. Remember that the e-book market size \( s_E \) must be greater than zero at \( p^*_E = 0 \) in order to make \( p^*_E = 0 \) economically sensible. Therefore, we have \( B_1 > 0 \). We find \( B_1 - C_1 = w_E^2 > 0 \).

As \( C_1 > 0 \), we find \( B_1 > 0 \). Since \( B_1 > 0 \), we have \( \pi_s \bigg|_{p^*_E = w_E} > \pi_s \bigg|_{p^*_E = 0} \). To verify the second order conditions, we derive the 2 × 2 Hessian matrix. We evaluate matrix \( H \) at optimal point \((p^*_E, p^*_D)\). We find \( H_{11} = \frac{\partial^2 \pi_s}{\partial p_E^2} = -\frac{1}{8} \frac{D_1}{m^2} \) and \( |H| = \frac{E_1}{8m^2} \) where \( D_1 \) and \( E_1 = D_1 - 8(b - w_E)^2 \). Therefore, we find \( D_1 > E_1 \). So in order to show \( H_{11} < 0 \) and \(|H| > 0 \), we only need to show \( E_1 > 0 \). We find \( s_E \bigg|_{p^*_E = 0} = \frac{C_1}{16m} \). We know \( s_E \bigg|_{p^*_E = 0} > 0 \). Hence we have \( E_1 > 0 \). So we verify that the optimal choice of e-book retail price \( p^*_E \) and e-reader price \( p^*_D \) in Proposition 1.

Proof of Proposition 2

We let \( \hat{\pi}_s \) denote the publisher’s total profit at the retailer’s optimal choice of prices \((p^*_E, p^*_D)\). First we will prove that \( 0 < w_E^* < \frac{b + c_m}{2} \).

We define \( f_s(w_E) = \frac{\partial \hat{\pi}_s}{\partial w_E} \). Notice that \( f_s \) is a cubic function of \( w_E \) and we find that \( \frac{\partial f_s}{\partial w_E} = -\frac{3}{8m} < 0 \). The potential optimal points are \( f_s(w_E^*) = 0 \), \( w_E^* = 0 \), or \( w_E^* = b \). We find that \( w_E^* = 0 \) can be removed from the candidate set as it is not difficult to show \( \hat{\pi}_s \bigg|_{w_E = 0} \) is less than the profit of not selling any e-book. Second, \( w_E^* = b \) can also be removed as it violates consumer’s IR constraint. Therefore, we focus on \( f_s(w_E^*) = 0 \). Next, we show \( f_s \bigg|_{w_E = 0} > 0 \). We derive \( f_s \bigg|_{w_E = 0} = \frac{A_1}{8m^2} \). It is not difficult to show
min $A_2 = 2((w_F - c_A - c_E)(b - p_F) + c_A b) > 0$. Therefore, we have $f_2 \bigg|_{w_e = 0} > 0$. Next we find $f_2 \bigg|_{w_e = (b - w_F) / 2} = -2c_A / 32m^2$. and $\min B_2 = 4(p_F - w_e + c_F(b - p_F) > 0$. Therefore, we have $f_2 \bigg|_{w_e = (b - w_F) / 2} < 0$. Meanwhile, we find that $\frac{\partial^2 f_2}{\partial w_e^2} \bigg|_{w_e = (b - w_F) / 2} = \frac{3b - c_F}{4m^2} > 0$.

Combining $f_2 \bigg|_{w_e = 0} > 0$, $f_2 \bigg|_{w_e = (b - w_F) / 2} < 0$, $\frac{\partial f_2}{\partial w_e} \bigg|_{w_e = (b - w_F) / 2} = \frac{b - c_F}{4m^2} > 0$, and $\frac{\partial^2 f_2}{\partial w_e^2} < 0$, we conclude that there is only one $w_e^* > 0$ such that $f_2(w_e^*) = 0$. Because $\frac{\partial f_2}{\partial w_e} \bigg|_{w_e = 0} < 0$, we find $\frac{\partial w_e^*}{\partial w_e} > 0$. Similarly, $\frac{\partial w_e^*}{\partial p_F}$ has the same sign as $\frac{\partial^2 f_2}{\partial w_e \partial p_F} \bigg|_{w_e = w_e^*}$. We then derive $\frac{\partial f_2}{\partial p_F} \bigg|_{w_e = w_e^*} = \frac{b - c_F}{4m^2} > 0$.

**Proof of Proposition 3**

Note that $\pi_R$ is a quadratic function for $p_D$ and $\frac{\partial^2 \pi_R}{\partial p_D^2} < 0$. Therefore, we solve $\pi_R = 0$ and then we find $p_D^*$ in Proposition 3.

**Proof of Proposition 4**

Proof of this proposition follows the procedure similar to Proposition 2. We let $\hat{R}$ denote the publisher’s total profit at the retailer’s optimal choice of e-reader price $p_D^*$. We define $f_4(p_e) = \frac{\partial^2 \hat{R}}{\partial p_e^2}$. Notice that $f_4$ is a cubic function of $p_e$ when $r = 1 / 2$ (in the case of $r = 1 / 2$, the proof will be straightforward). Next, we show $f_4 \bigg|_{p_e = 0} > 0$. We derive $f_4 \bigg|_{p_e = 0}$ and follow the similar steps as the proof of Proposition 2. We find that $\min f_4 \bigg|_{p_e = 0} = \frac{2b(1-r)((w_F - c_p - c_A)(b - p_F) + 2c_A)}{8m^2} > 0$. So we have $f_4 \bigg|_{p_e = 0} > 0$.

Next we find $f_4 \bigg|_{p_e = p_e^*} = \frac{(c_A - 2c_p r - b + rb)A}{32m^2(1-r)^2}$. and $\min A_4 = 4(1-r)(p_F - p_F r + c_p - w_F)(b - p_F)$. It is not difficult to show $c_A - 2c_p r - b + rb < 0$. Therefore, $f_4 \bigg|_{p_e = p_e^*} < 0$ when $r < \frac{p_F - w_F + c_p}{p_F}$. Meanwhile, we find that $\frac{\partial f_4}{\partial p_e} \bigg|_{p_e = p_e^*} = -\frac{3c_A - 2c_p r - b + rb}{m^2} > 0$. Combining $f_4 \bigg|_{p_e = 0} > 0$, $f_4 \bigg|_{p_e = p_e^*} < 0$, and $\frac{\partial f_4}{\partial p_e} \bigg|_{p_e = p_e^*} > 0$ and the fact that $f_4$ is a cubic function, we conclude that there exists only one $p_e^*$ in $(0, p_e^*)$ satisfying $f_4(p_e^*) = 0$. Therefore, we have $p_e^* < p_e$ when $r < \frac{p_F - w_F + c_p}{p_F}$.

**Proof of Proposition 5**

Having shown the result $0 < p_e^* < p_e$ in Proposition 4, we will next prove that $\frac{\partial^2 \hat{R}}{\partial p_e^2} < 0$ and $\frac{\partial^2 \hat{R}}{\partial p_F^2} < 0$. Has the same sign as $\frac{\partial^2 \hat{R}}{\partial p_e \partial p_F}$. We derive $\frac{\partial^2 \hat{R}}{\partial p_e \partial p_F} \bigg|_{p_e = p_e^*} = \frac{(b - p_F)(p_e - 2p_F r - b + rb)}{4m^2}$. It is not difficult to show $p_e - 2p_F r - b + rb < 0$. Hence, we find $\frac{\partial^2 \hat{R}}{\partial p_e^2} < 0$. Similarly, $\frac{\partial^2 \hat{R}}{\partial p_F^2}$ has the same sign as $\frac{\partial^2 \hat{R}}{\partial p_e \partial p_F} \bigg|_{p_e = p_e^*}$. We then derive
Next, we will prove . We define (i.e., we treat as a function of in the wholesale model’s SPNE by . We align the two first order conditions \( \frac{\partial^2 \hat{\pi}_p}{\partial \hat{p}_E \partial \hat{c}_E} \big|_{\hat{p}_E = \hat{c}_E} = \frac{(1-r)(b-p_E)(\hat{p}_E - p_E)}{2m t} \). Given \( 0 < p'_E < \hat{p}_E \), we have \( \frac{\partial^2 \hat{\pi}_p}{\partial \hat{p}_E \partial \hat{c}_E} \big|_{\hat{p}_E = p'_E} > 0 \). Therefore, we proved \( \hat{\pi}_p > 0 \).

**Proof of Proposition 6**

In this subsection of proof, we denote the publisher’s profit in the agency model’s SPNE by \( \hat{\pi}^*_p \). We denote the publisher’s profit in the wholesale model’s SPNE by \( \hat{\pi}^*_p \). We align the two first order conditions \( \frac{\partial^2 \hat{\pi}^*_p}{\partial \hat{p}_E \partial \hat{c}_E} = 0 \) and \( \frac{\partial^2 \hat{\pi}^*_p}{\partial \hat{w}_E \partial \hat{c}_E} = 0 \). We eliminate a common term \( m(t-c_D) - w_E p_E + c_E b + p_E^2/2 - c_E p_E + w_E b \) and then obtain a new equation \( f(p^*_E, w^*_E) = 0 \). Since \( w^*_E = p^*_E \) in the wholesale model’s SPNE, in order to prove \( p^*_E > p^*_w \), we just need to prove \( p^*_E > w^*_w \). We define \( w^*_w = kp^*_E \) and substitute it into \( f(p^*_E, w^*_w) = 0 \). This transforms \( f(p^*_E, w^*_w) = 0 \) into \( f(k) = 0 \). We define \( f(k) = m t - f(k) \). Then we just need to prove that there exists one and only one root of \( f(k) \) in \([0, 1]\). We will prove it by proving \( f'_9|_{k=0} < 0 \), \( f'_9|_{k=1} > 0 \), and \( \partial f_9 / \partial k > 0 \) in \( 0 < k \leq 1 \).

In the following, for simplicity, we denote \( p^*_E \) by \( p_E \) and denote \( w^*_w \) by \( w_E \).

First we will prove \( \partial f_9 / \partial k > 0 \) in \( 0 < k \leq 1 \). We derive \( \frac{\partial^2 f_9}{\partial k^2} = -\frac{1}{4} \left( \frac{p_E - b - c_A + 2 p_E - 2 p_E^2}{(b-c_A)^2} \right) \) and \( f_9(p_E) \) is a cubic function of \( p_E \) with other parameters. It is not difficult to show \( f_9(p_E) < 0 \) when \( p_E < p_E = (b + c_A)/(1-r) / 2 \). Next we will prove \( f_9(p_E) \). In Proposition 2.3, we have proved \( w_E < w_E = (b + c_A) / 2 \). Since we have \( w_E = k p_E \), we have \( p_E < p^*_E = (b + c_A) / (2 k) \). We derive \( f_9(p_E) = \frac{\partial^2 f_9}{\partial p_E^2} = -48 k^3 p_E + 18 k^2 c_A + 30 k^2 b \) and \( f_9(p_E) = \frac{\partial f_9}{\partial p_E} = -24 k^3 p_E + 2(9 k^2 c_A + 15 k^2 b) p_E - 12 k c_A b - 9 k b^2 - 3 k c_A^2 \). We first find \( f_9|_{p_E = p_E} = 6 k^2 (b - c_A) > 0 \). Second, when \( 0 < k \leq 1 \), we find \( f_9|_{p_E = p_E} = 6 k^2 (5 b + 3 c_A) > 0 \). Therefore, \( f_9(p_E) > 0 \). Next, we find \( f_9|_{p_E = p_E} = 0 \) and \( f_9|_{p_E = p_E} = 0 \). Together with \( f_9(p_E) = \partial f_9 / \partial p_E > 0 \), we find \( f_9(p_E) < 0 \). Therefore, in order to show \( f_9(p_E) > 0 \), we just need to prove \( f_9|_{p_E = p_E} > 0 \). We find \( f_9|_{p_E = p_E} = \frac{1}{4} (b - c_A) f_14 \) and \( \min f_14 = 4 (p_F - w_E + c_p)(b - p_F) > 0 \). Therefore, we proved \( f_9(p_E) > 0 \). Together with \( f_9(p_E) < 0 \), we proved \( \partial f_9 / \partial k > 0 \) in \( 0 < k \leq 1 \).

Next, we will prove \( f'_9|_{k=0} < 0 \). We define \( f_9(p_E) = f_9|_{k=0} \) (i.e., we treat \( f_9 \) as a function of \( p_E \)). Notice that \( f_9(p_E) \) is a cubic function of \( p_E \). We derive \( f_9|_{p_E = p_E} = -\frac{1}{4} \frac{c_A r b (b - p_E)(w_E - c_A) + c_A p_E}{c_A + b} \). We find that \( f_9|_{p_E = p_E} < 0 \). Next, we find \( f_9|_{p_E = p_E} = \frac{1}{32} \frac{(c_A^2 - 2 c_A b - b + r b) f_{18}}{(1-r)^2} \) and \( \min f_{18} = -4(1-r)(p_E r - c_A + w_E - p_E)(b - p_E) \). It is not difficult to show when \( r < p_E - w_E + c_A \), we have \( f_9|_{p_E = p_E} < 0 \). We define \( f_9(p_E) = \frac{\partial^2 f_9}{\partial p_E^2} \). We find that \( f_9(p_E) \) is a linear function of \( p_E \). We find that \( f_9(p_E) > 0 \) in \( p_E \in (0, \hat{p}_b) \) when \( r \leq \frac{3}{4} \). We also find that when \( 0 < r < 1 \) we have \( f_9|_{p_E = p_E} > 0 \). Therefore we prove \( \partial^2 f_9 / \partial p_E^2 > 0 \) in \( p_E \in (0, \hat{p}_b) \) when \( r \leq \frac{3}{4} \).

Next, we will prove \( f_9|_{k=1} > 0 \). We derive \( f_9(r) = f_9|_{k=1} \) as a quadratic equation of \( r \). We derive \( f_9|_{r=0} = 0 \) and \( \partial^2 f_9 / \partial r^2 = -(b - w_E)(b - 2 w_E) w_E \). In order to show \( f_9|_{k=1} > 0 \), we need \( w_E < b / 2 \) and \( f_9|_{r=1} > 0 \). We derive \( f_9|_{r=1} = f_9|_{r=0} \).
where \( f_{21}(w_e) \) is a cubic function of \( w_e \). We find that \( f_{21} = 2c_d^2(b - c_d)^2 > 0 \) and \( f_{21} = 1/8b^2c_d(2c_d + b) > 0 \). We also find that \( \partial^2 f_{21} / \partial w^2 > 0 \) and \( \partial^3 f_{21} / \partial w^3 < 0 \) when \( p_F < b \). Therefore, we find \( f_{21} > 0 \) when \( c_d < w_e < b/2 \). Therefore, to make \( \partial^3 f_{21} / \partial w^3 > 0 \), we only need \( c_d < w_e < b/2 \). Following the same procedure of proving \( 0 < w_e < (b + c_d)/2 \) in Proposition 2, we find that a sufficient condition for \( c_d < w_e < b/2 \) is \( w_F - c_d < p_F/2 \) and \( c_d < p_F/4 \). Therefore, we prove \( p_e^* > p_e^* \) when

\[
n_e^* - p_e^* = -\frac{1}{4}(p_e^* - p_e^*)^2 + \frac{1}{2}p_e^*(b - p_e^*).
\]

Since we find that when \( p_e^* > p_e^* \), we have \( p_e^* < p_e^* \).

**Proof of Proposition 7 and Proposition 8**

For the wholesale mode, we solve the retailer’s constraint optimization problem using the Lagrange method. From the first order condition, we obtain

\[
p_{D1}^* = \frac{t + c_d}{2} + \frac{A}{(4m)} + \frac{\theta_d(b - p_e)^2(1 - k)(2 - a)}{(4m)} + \frac{\theta_d(b - p_e)^2}{(4m)} - \frac{\theta_d(b - p_e)(1 - k)a}{(4m)} \] where

\[
\theta_d = a \theta_d + (1 - a) \theta_d , \quad c_d = ac_d + (1 - a)c_d , \quad A = 2(b - p_e)(p_F - w_F - c_F) + k(b - p_e)^2 - 2(p_F - w_F)(b - p_e) - (b - p_e)^2.
\]

Consider \( p_{D1}^* \) and \( p_{D2}^* \) as functions of \( p_e \). We plug them back to the retailer’s profit function \( \pi_R \). We define that

\[
g_e(p_e) = \left((p_e - w_F)q_F D + (p_{D1}^* - c_d)q_{D1} D\right)q_{F1} D + \left(p_F - w_F - c_F\right)q_{F2} D, \quad g_F(p_e) = \left((p_e - w_F)q_{E1} F + \left(p_{D2}^* - c_d\right)q_{F2} D + (p_F - w_F - c_F)q_{F1} D\right)q_{F1} D.
\]

The \( \pi_e \) can be expressed by \( \pi_e = ag_e + (1 - a)g_F \). Following the similar steps of the proof in Proposition 2, it is not difficult to show that (i) \( \partial g_e / \partial p_e = 0 \) can only be attained at \( p_{E1} > w_F \) when \( p_{E1}^* > w_F \), (ii) \( \partial g_e / \partial p_e = 0 \) can only be attained at \( p_{E2}^* > w_F \), (iii) \( \partial^2 g_e / \partial p^2 e < 0 \) and \( \partial^2 g_e / \partial p^2 e < 0 \). As \( \pi_e \) is a convex combination of \( g_e \) and \( g_F \), we find \( p_e^* > w_F \). For the agency model, it is straightforward to show the results presented in Proposition 8 through solving the first order conditions using the Lagrange method.