Appendix A

Proofs

Proof of Proposition 1: We first show that for all $t$, $\frac{1}{\lambda} \neq \beta_t$. Plugging Equations (3) and (6) into Equation (7) gives

$$dP_t = \lambda \beta_t dP_t + \lambda \alpha_W + \lambda \alpha_t (\tilde{v} - P_t)dt$$

If $\frac{1}{\lambda} = \beta_t$, then $\alpha_W = -\alpha_t (\tilde{v} - P_t)dt$ holds for all $\sigma > 0$ and $\alpha_t > 0$. Mathematically, it incorrectly implies that the Brownian motion is determined by a drift in time. From a practical point of view, it incorrectly implies that informed traders bring only noise into the market.

When $\frac{1}{\lambda} \neq \beta_t$, we have

$$dP_t = \frac{\lambda \sigma}{1 - \lambda} (\tilde{v} - P_t)dt + \frac{\lambda \sigma}{1 - \lambda} dW_t$$

(22)

Note that Equation (22) is under filtration $F_t \equiv F_v \cup \sigma(t)$. For a given $F_t$, taking the conditional expectation of Equation (22) yields

$$dP_t = \frac{\lambda \sigma}{1 - \lambda} dW_t$$

(23)

This is a stochastic differential equation of $P_t$ under filtration $F_t$. To examine the properties of the price process, we need to apply the filtering lemma by Lipster and Shiryayev (1977), which helps answer the following question: Given the observations of the stochastic process $P_t$, what is the best estimate of the state $\tilde{v}$ based on these observations?

First, let $V_t = E(\tilde{v}|F_t)$ and consider the filtering of $\tilde{v}$ with respect to $\{F_t\}_{t=1}^T$. By the filtering lemma, we have

$$dV_t = E(\tilde{v}|F_t) dt + \frac{\lambda \alpha_t}{1 - \lambda \beta_t} \left( \frac{1 - \lambda \beta_t}{\lambda \sigma} \right) dW_t$$

thus
\[ dV_t = \delta(t) \frac{\sigma}{\tau} dW_t \]  

(24)

where

\[ \delta(t) = E[(v - V_t) | F_t] \]

satisfies the following one-dimensional Riccati differential equation:

\[ \frac{d\delta(t)}{dt} = -\delta(t) \left( \frac{\lambda_\sigma}{(1 - \lambda_\beta)} \sigma \right) \left( \frac{1 - \lambda_\beta}{\lambda_\sigma} \right) \delta(t) \]

That is,

\[ \frac{d\delta(t)}{dt} = -\frac{\alpha^2}{\sigma^2} \delta^2(t) \]

with initial value

\[ \delta(0) = \sigma^2 \]

The solution to this equation is

\[ \delta(t) = \left[ \sigma^2 + \int_0^t \frac{\sigma}{\tau} ds \right]^{-1} \]

(25)

Plugging Equation (25) into Equation (24) and using the semi-strong efficiency condition gives

\[ dP_t = \frac{\sigma}{\tau} \left[ \sigma^2 + \int_0^t \frac{\sigma}{\tau} ds \right]^{-1} dW_t \]

(26)

Comparing coefficients of Equation (23) and Equation (26) yields

\[ \left[ \sigma^2 + \int_0^t \frac{\sigma}{\tau} ds \right]^{-1} \frac{\sigma}{\tau} = \frac{\lambda_\sigma}{1 - \lambda_\beta} \]

(27)

Thus,

\[ \frac{\sigma}{\tau} \left[ \sigma^2 + \int_0^t \frac{\sigma}{\tau} ds \right] = \frac{1}{\lambda} - \beta \]

(28)

Since \( \alpha \) is strictly positive, we can see that when the market is semi-strong efficient the depth of the market \( \frac{1}{\alpha t} \) is always greater than \( \beta \).

Equation (27) can be rewritten as

\[ \left( \frac{\sigma}{\tau} \right)^2 \left[ \sigma^2 + \int_0^t \frac{\sigma}{\tau} ds \right] = \left( \frac{\lambda_\sigma}{1 - \lambda_\beta} \right)^2 \]

Integrating the above equation with respect to \( dt \) yields

\[ \alpha_t = \frac{\lambda_\sigma \sigma^2}{\left( \sigma^2 - \int_0^t \left( \frac{\lambda_\sigma}{1 - \lambda_\beta} \right)^2 ds \right)(1 - \lambda_\beta)} \]

(29)

Again, since \( \alpha_t \) is strictly positive, it is easy to see that for all \( t \in [0, 1] \)

\[ \sigma^2 - \int_0^t \left( \frac{\lambda_\sigma}{1 - \lambda_\beta} \right)^2 ds > 0 \]

Q.E.D.
Proof of Proposition 2: By Schwartz inequality and the constraint, we now that

\[
\int_0^t \frac{\lambda_s}{1 - \lambda_t} ds \leq \left( \int_0^t \left( \frac{\lambda_s}{1 - \lambda_t} \right)^2 ds \right)^{\frac{1}{2}} \leq \frac{\sigma}{\sigma_t}
\]

The equality holds if and only if for any \( t \in [0, 1], \)

\[
\frac{\lambda_s \sigma}{1 - \lambda_t} = \frac{\sigma}{\sigma_t}
\]

(30)

Q.E.D.

Proof of Theorem 3: Equation (16) can be obtained directly from Equation (30). Equation (17) is obtained by plugging Equation (16) into Equation (29). Equations (27) and (25) combined yields Equation (18). Finally, Equation (19) is obtained by combining Equations (15) and (30).

Q.E.D.

Proof of Proposition 4: Under the assumption in our model, the profit earned by uninformed traders can be expressed by

\[
E \left[ (1 - \int_0^t (v_t - P_t) dX_t(s)) \right]
\]

and

\[
E \left[ (1 - \int_0^t (v_t - P_t) dX_t(s)) \right]
= E \left[ (1 - \int_0^t (v_t - P_t)) (\beta_t dP_t + \alpha_t dW_t) \right]
= E \left[ (1 - \int_0^t (v_t - P_t)) \left( \frac{\beta_t \lambda_t \sigma}{1 - \lambda_t} (v_t - P_t) ds + \left( \frac{\lambda_t \beta_t \sigma}{1 - \lambda_t} \right) dW_t \right) \right]
= E \left[ (1 - \int_0^t (v_t - P_t) ) \frac{\sigma}{1 - \lambda_t} dW_t \right] + E \left[ (1 - \int_0^t (v_t - P_t)) \frac{\sigma}{1 - \lambda_t} dW_t \right]
= \int_0^t \frac{\beta_t \lambda_t \sigma}{1 - \lambda_t} \alpha_t ds + E \left[ (1 - \int_0^t (v_t - P_t)) \left( \frac{\lambda_t \beta_t \sigma}{1 - \lambda_t} \right) dW_t \right]
= \sigma \int_0^t \beta_t ds - E \left[ (1 - \int_0^t \lambda_t dX_t(q)) \frac{\sigma}{1 - \lambda_t} dW_t \right]
= \sigma \int_0^t \beta_t ds - E \left[ (1 - \int_0^t \lambda_t dX_t(q)) \frac{\lambda_t \sigma}{1 - \lambda_t} dW_t \right]
= \sigma \int_0^t \beta_t ds - E \left[ (1 - \int_0^t \sigma \frac{\lambda_t \beta_t \sigma}{1 - \lambda_t} dW_t \right]
= \sigma \int_0^t \beta_t ds - \sigma (v_t + \beta_t) ds
= -\sigma \int_0^t \sigma ds
= -\sigma \sigma_t
\]

The result is obtained from Equations (7), (22), (16), and (18), and the transformation relation between the Itô and the (1–) stochastic integration. The last equation assumes that \( \sigma \) is not a function of \( t \). If \( \sigma \) is indeed a function of \( t \), the result is not changed: \( \int_0^t \sigma \) simply measures the average variance of noise up to time \( t \). Whether \( \sigma \) is a function of time does not change any of our results.

Q.E.D.
Proof of Theorem 5: Without loss of generality, we suppose \( P_0 = 0 \). The second moment of the informed trader’s profits is

\[
E \left[ (1-t) \int_0^t (\bar{v} - P_t) dX_t(t) \right]^2 = E \left[ (1-t) \int_0^t (\bar{v} - P_t)^2 \frac{1}{\sigma_r} dW_t \right]^2
\]

\[
= E \left[ \int_0^t (\bar{v} - \bar{v}_t - (1-t) \int_0^t \frac{\sigma_r}{1-s} dW_s)^2 \frac{\sigma_r}{1-t} dt \right]^2
\]

\[
= E \left[ \int_0^t (1-t) \left( \bar{v} - \int_0^t \frac{\sigma_r}{1-s} dW_s \right)^2 \frac{\sigma_r}{1-t} dt \right]^2
\]

\[
= E \left[ \int_0^t (1-t)^2 v^2 \frac{\sigma_r}{\sigma_v} dt - 2 \int_0^t (1-t) \frac{\sigma_r}{\sigma_v} \left( \int_0^t \frac{\sigma_r}{1-s} dW_s \right) dt + \int_0^t (1-t) \left( \int_0^t \frac{\sigma_r}{1-s} dW_s \right)^2 \frac{\sigma_r}{\sigma_v} dt \right]^2
\]

Defining the first term by \( A_1 \),

\[
A_1 = \int_0^t (1-t)^2 \frac{\sigma_r}{\sigma_v} dt = \frac{\sigma_r}{2\sigma_v} \bar{v}^2
\]

Integrating by parts (stochastic integration, generalized Itô formula), we can have

\[
\int_s^t \frac{1}{1-s} dW_s = \left. \frac{W_r}{1-s} \right|_s^t - \int_s^t \frac{W_r}{1-s} ds = \frac{W_r}{1-t} - \int_s^t \frac{W_r}{1-s} ds
\]

By interchangeability of ordinary Riemann integration, we can calculate

\[
A_2 = \int_0^t (1-t)^2 \left( \int_0^t \frac{\sigma_r}{1-s} dW_s \right) \frac{\sigma_r}{\sigma_v} dt
\]

\[
= 2 \bar{v} \sigma_r \int_0^t (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right) dt
\]

\[
= 2 \bar{v} \sigma_r \int_0^t (1-t) \left( \int_0^t \frac{1}{1-s} dW_s - \int_0^t \frac{W_r}{1-s} ds \right) dt
\]

\[
= 2 \bar{v} \sigma_r \left[ \int_0^t W_r dt - \int_0^t \int_0^t (1-t) \frac{W_r}{1-s} ds dt \right]
\]

\[
= 2 \bar{v} \sigma_r \left[ \int_0^t W_r dt \left( 1 - \frac{1}{1-t} + \frac{1}{2} \frac{1}{1-t} \right) dt \right]
\]

\[
= 2 \bar{v} \sigma_r \int_0^t W_r dt
\]

And the last term

\[
A_3 = \int_0^t (1-t) \left( \frac{\sigma_r}{\sigma_v} \left( \int_0^t \frac{\sigma_r}{1-s} dW_s \right)^2 dt
\]

\[
= \sigma_r \int_0^t (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt
\]

The informed trader’s variance of the profit is
\[
E[(A_1 + A_2 + A_3)^2 - (E[\tilde{P}(1)])^2] \\
= E[A_1^2 + A_2^2 + A_3^2 + 2A_1A_2 + A_1A_3 + 2A_2A_3] - E[\tilde{P}(1)]^2 \\
\]
\[
= \sigma^4 + 2\sigma^2 \sigma^2 E[\tilde{V}^2] + \sigma^4 E\left[\int_0^t (1-t)\left(\int_0^t \frac{1}{1-s}dW_s\right)^2 dt\right] + \frac{1}{2} \sigma^4 E[\tilde{V}^2] - \sigma^2 \sigma^2 \\
\]
\[
= \sigma^4 \left[ \frac{1}{4} + E\left[\int_0^t W_s dt\right]^2 + \left[\int_0^t (1-t)\left(\int_0^t \frac{1}{1-s}dW_s\right)^2 dt\right]^2 + \frac{1}{2} - 1 \right] \\
= \frac{1}{4} \sigma^4 \sigma^2
\]

We have used the assumption that \( \tilde{V} \) is independent of the Brownian motion \( W_t \), and the expectation of \( \tilde{V} \) is zero (i.e., \( \tilde{P} \equiv E[\tilde{V}] = 0 \)), and the last equality is obtained from results in Appendix B.

We continue to calculate the variance of the uninformed traders’ profits. For simplicity, we suppose that \( \beta_t \) is a constant over \( t \), denoted by \( \beta \), the second moment of the uninformed traders’ profit is

\[
E\left[(1-t)\int_0^t \tilde{V}^2 dt\right] \\
= E\left[(1-t)\int_0^t \tilde{V}^2 - \beta F \tilde{V} + \alpha dW_t\right] \\
= E\left[(1-t)\int_0^t \tilde{V}^2 - \beta F \tilde{V} + \alpha dW_t\right] \\
= E\left[(1-t)\int_0^t \tilde{V}^2 - \beta F \tilde{V} + \alpha dW_t\right] \\
= E\left[\int_0^t \tilde{V}^2 dt\right] + 2E\left[\int_0^t \frac{\tilde{P}}{1-t} \tilde{V} dt\right] \left(1-t\right)\int_0^t \tilde{V}^2 dt + \beta \sigma \alpha dW_t \\
+ E\left[(1-t)\int_0^t \tilde{V}^2 dt\right] (\beta \sigma + \alpha) dW_t
\]

The first term

\[
B_t = E\left[\beta \tilde{V} \int_0^t \frac{1}{1-t} \tilde{V} dt\right] \\
= \beta^2 \sigma^2 \sigma^2 \sigma^2 \left[ \frac{5}{4} + E\left[\int_0^t W_s dt\right]^2 + E\left[\int_0^t (1-t)\left(\int_0^t \frac{1}{1-s}dW_s\right)^2 dt\right]^2 \right] \\
= \frac{7}{4} \beta^2 \sigma^4
\]

This last equality is obtained from Appendix B.

The second term
\[B_2 = 2\beta(\beta\sigma + \alpha) E\left[\left(\int_0^t \frac{1}{1-t} (\tilde{v} - P_t)^2 dt\right)(1-t)\int_t^{1-t} (\tilde{v} - P_t) dW_t\right]
\]
\[= 2\beta(\beta\sigma + \alpha) E\left[\left(\int_0^t (\tilde{v} - P_t) dW_t + \int_0^t \sigma_t dt\right)(1-t)\int_t^{1-t} (\tilde{v} - P_t) dW_t - \sigma_t\right]
\]
\[= 2\beta(\beta\sigma + \alpha) E\left[\left(\int_0^t (\tilde{v} - P_t) dW_t + \int_0^t \sigma_t dt\right)(1-t)\int_t^{1-t} (\tilde{v} - P_t) dW_t - \sigma_t\right]
\]
\[= 2\beta^2(\beta\sigma + \alpha) E\left[\left(\int_0^t (\tilde{v} - P_t) dW_t + \int_0^t \sigma_t dt\right)^2\right]
\]
\[= \frac{7}{4}\beta^2\sigma^2 + \frac{7}{2}\beta(\beta\sigma + \alpha)^2 - \sigma^2\sigma^2
\]

The third term

\[B_3 = (\beta\sigma + \alpha)^2 E\left[\left(\int_0^t (\tilde{v} - P_t) dW_t\right)^2\right]
\]
\[= (\beta\sigma + \alpha)^2 E\left[\left(\int_0^t (\tilde{v} - P_t) dW_t + \int_0^t \sigma_t dt\right)^2\right]
\]
\[= (\beta\sigma + \alpha)^2 E\left[\int_0^t (\tilde{v} - P_t)^2 dt + \sigma^2\sigma^2\right]
\]
\[= \frac{3}{2}\sigma^2(\beta\sigma + \alpha)^2
\]

Here we have used the isometric property of the stochastic integral. The uninformed traders’ variance of profits is, therefore,

\[B_1 + B_2 + B_3 - \sigma^2\sigma^2 = \frac{7}{4}\beta^2\sigma^2 + \frac{7}{2}\beta(\beta\sigma + \alpha)^2 - \sigma^2\sigma^2
\]

\[= \frac{1}{2}\beta^2\sigma^2 + \frac{1}{2}\sigma^2\sigma^2
\]

Q.E.D.

Reference


Appendix B

Calculation of Expectations

Here we show how to calculate some expectations useful for Appendix A. First, we calculate \( E\left[\int W_t dt\right]^2 \):
\[
E \left[ \int_0^t W_s \, dt \right]^2 = E \left[ \int_0^t W_s \, dt \int_0^t W_s \, ds \right]
\]
\[
= \int_0^t \int_0^t E [W_s W_t] \, ds \, dt
\]
\[
= \int_0^t \int_0^t \min(s, t) \, ds \, dt
\]
\[
= \int_0^t \left( \int_s^t dt + \int_s^t ds \right) \, dt
\]
\[
= \frac{1}{3} \left( \frac{s^2}{2} + s(1-s) \right) \, ds = \frac{1}{3}
\]

In the following, we calculate

\[
\Gamma = E \left[ \int_0^1 (1-t) \left( \int_0^t (1-s) \, dW_s \right)^2 \, dt \cdot \int_0^1 (1-t) \int_0^t \frac{1}{1-s} \, dW_s \, dW_t \right]
\]

Let

\[
I_t = \int_0^t \frac{1}{1-s} \, dW_s
\]
\[
X_t = \int_0^t (1-t) I_s^2 \, dt
\]
\[
Y_t = \int_0^t (1-t) I_s \, dW_s
\]

where \( I_t \) and \( Y_t \) are martingales. What we want is

\[
X_t Y_t
\]

Integrating by parts,

\[
X_t Y_t = \int_0^t X_s \, dY_s + \int_0^t Y_s \, dX_s
\]

Since \( Y_t \) is a martingale,

\[
E[X_t Y_t] = E\left[ \int_0^t Y_s \, dX_s \right]
\]
\[
= E\left[ \int_0^t Y_s (1-\tau) I_{\tau}^2 \, d\tau \right]
\]
\[
= \int_0^t (1-\tau) E[Y_s I_{\tau}^2] \, d\tau
\]

Integrating by parts, we have

\[
Y_t I_{\tau}^2 = \int_0^\tau I_{\tau}^2 \, dY_s + \int_0^\tau Y_s \, dI_{\tau}^2 + \frac{1}{2} \int_0^\tau d < Y_s I_{\tau}^2 >
\]

where \( < X_s, Y_s > \) denotes the quadratic variation process of \( X \) and \( Y \).

\[
dI_{\tau}^2 = 2I_{\tau} \, dI_{\tau} + d < I_s I_{\tau} > = 2I_{\tau} \, dI_{\tau} + \frac{1}{(1-\tau)} \, dt
\]

therefore
$E[Y, t^2] = \mathbb{E} \left[ \int_0^t Y \cdot \frac{1}{1-t} \, dt + \frac{1}{2} \int_0^t \int_0^t (1-t) I_{s} \cdot 2t \cdot \frac{1}{1-t} \, dt \right]$

$= \int_0^t E[I_s] \, dt = \int_0^t \left( \int_0^t \frac{1}{1-s} \, ds \right) \, dt$

$= \int_0^t \left( \frac{1}{1-t} - 1 \right) \, dt = -\ln(1-t) - t$

$E[X, Y] = \int_0^t (1-t) E[Y, t^2] \, dt$

$= -\int_0^t x \ln(x) \, dx - \int_0^t (1-t) \, dt = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Hence we obtain

$\Gamma = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

Next, we calculate $\mathbb{E} \left[ \int_0^t (1-t) \left( \int_0^t \frac{1}{1-s} \, ds \right)^2 \, dt \right]$. Using the same notation as above, what we want is $E[X_t^2]$. Integrating by parts,

$X, X_t = 2 \mathbb{E} \left[ X_t dX_t \right]$,

Hence,

$E[X_t^2] = 2 \mathbb{E} \left[ \int_0^t X_t dX_t \right]$

$= 2 \mathbb{E} \left[ \int_0^t X_t (1-t) I_t \, d\tau \right]$

$= 2 \int_0^t (1-t) E[X_t, t^2] \, d\tau$

Integrating by parts, we have

$X_t, t^2 = \int_0^t I_t \, dX_t + \int_0^t X_t dI_t + \frac{1}{2} \int_0^t d <X_t, I^2>$,

$= \int_0^t (1-t) I_t \, dt + \int_0^t X_t dI_t$

where $<X, Y>$, denotes the quadratic variation process of $X$ and $Y$.

$dl_t^2 = 2l_t dl_t + d <l_t, l_t> = 2l_t dl_t + \frac{1}{(1-t)} \, dt$

therefore

$E[X_t, t^2] = E \left[ \int_0^t (1-t) I_t \, dt + \int_0^t X_t \frac{1}{(1-t)} \, dt \right]$

$= \int_0^t (1-t) E[I_t] \, dt + \int_0^t E[X_t] \frac{1}{(1-t)} \, dt$

$= \int_0^t (1-t) \frac{3t^2}{2} \frac{1}{(1-t)} \, dt + \frac{1}{2} \int_0^t t^2 \frac{1}{(1-t)} \, dt$

$= -3\left(t + \frac{1}{2}\right) - 3\ln(1-t) + \frac{1}{2} \left(\tau + \ln(1-t) + \frac{1}{1-t} - 1\right)$

$= -\frac{5}{2} \left(\tau + \ln(1-t) + \frac{1}{2} \left(\frac{1}{1-t} - 1 - 3\tau^2\right)\right)$
Here we have used the result that \( I_i \sim N(0, \frac{1}{\theta_i}) \), then \( E[I_i^2] = \frac{\theta_i}{(1-\theta_i)^2} \) and \( E[I_i] = \frac{\theta_i}{1-\theta_i} \).

\[
E[X_i^2] = 2 \int_0^1 (1-\tau)E[I_i^2]d\tau = -\int_0^1 5x \ln(x)dx - 5' \int_0^1 (1-\tau)dt + \int_0^1 (1-\tau)dt - \int_0^1 3' (1-\tau)d\tau
\]

\[
= \frac{5}{4} - \frac{5}{6} + \frac{3}{4} = \frac{1}{6}
\]

**Appendix C**

**Proof of Theorem 7**

Inserting the uninformed traders’ new demand Equation (21) to the pricing rule Equation (7), we can obtain

\[
dP = \left[ \frac{\hat{\lambda}_\alpha}{1-\lambda_\beta} \bar{v} + \frac{\hat{\lambda}_\gamma}{1-\lambda_\beta} (\bar{v} + \epsilon) - \frac{\hat{\lambda}_1 (\alpha_1 + \gamma_1)}{1-\lambda_\beta} P_i \right] dt + \frac{\hat{\lambda}_\sigma}{1-\lambda_\beta} dW_i,
\]

The logic of deriving the result is the same as before. However, since we have one additional dimension of uncertainty coming from \( \epsilon \), the filtering process needs to deal with the vector \( \left( \bar{v}, \bar{v} + \epsilon \right) \).

Consequently, the deviation of the price from the liquidation value at time \( t \), \( \delta(t) \), is a matrix

\[
\delta(t) = \begin{pmatrix}
\delta_1(t) & \delta_2(t) \\
\delta_3(t) & \delta_4(t)
\end{pmatrix}
\]

with \( \delta_1(t) = E[\bar{v} - E[\bar{v}^2]] \), \( \delta_2(t) = E[(\bar{v} + \epsilon - E[\bar{v} + \epsilon])^2] \), \( \delta_3(t) = E[(\bar{v} + \epsilon - E[\bar{v} + \epsilon])^2] \), and \( \delta_4(t) = E[(\bar{v} + \epsilon - E[\bar{v} + \epsilon])^2] \).

The variance-covariance matrix can be derived as

\[
\begin{pmatrix}
\delta_1(t) & \delta_2(t) \\
\delta_3(t) & \delta_4(t)
\end{pmatrix} = \left[ \begin{pmatrix}
\frac{\sigma_1^2}{\sigma_\epsilon^2} & \frac{\sigma_1^2}{\sigma_\epsilon^2} \\
\frac{\sigma_2^2}{\sigma_\epsilon^2} & \frac{\sigma_2^2}{\sigma_\epsilon^2}
\end{pmatrix} \right] \begin{pmatrix}
\delta_1(t) & \delta_2(t) \\
\delta_3(t) & \delta_4(t)
\end{pmatrix} \quad \text{for} \quad \text{Equation (31)}
\]

where the symbol \( \prime \) denotes the transpose of the matrix.

Equation (31) is a matrix Riccati differential equation with initial value

\[
\delta(0) = \begin{pmatrix}
\delta_1(0) & \delta_2(0) \\
\delta_3(0) & \delta_4(0)
\end{pmatrix} = \begin{pmatrix}
\sigma_1^2 & \sigma_2^2 \\
\sigma_1^2 + \sigma_\epsilon^2 & \sigma_2^2 + \sigma_\epsilon^2
\end{pmatrix}
\]

The solution to the equation is

\[
\delta(t) = \left( \delta(0) \right)^{-1} + \left[ \int_0^t \frac{\sigma_1^2}{\sigma_\epsilon^2} ds \quad \int_0^t \frac{\sigma_2^2}{\sigma_\epsilon^2} ds \quad \int_0^t \frac{\sigma_1^2}{\sigma_\epsilon^2} ds \quad \int_0^t \frac{\sigma_2^2}{\sigma_\epsilon^2} ds \right]^{-1}
\]

After calculation, we obtain that

\[
\delta(0) = \begin{pmatrix}
\frac{\sigma_1^2 + \frac{1}{\sigma_\epsilon^2}}{\sigma_\epsilon^2} & -\frac{1}{\sigma_\epsilon^2} \\
-\frac{1}{\sigma_\epsilon^2} & \frac{\sigma_2^2 + \frac{1}{\sigma_\epsilon^2}}{\sigma_\epsilon^2}
\end{pmatrix}
\]
For the conditional expectation, we have

Similarly, the uninformed trader with imprecise information tries to maximize her profit at time 1. The maximization problem is independent of the feedback intensity.

A10

The informed trader’s expected profit at time 1 is

Overall, this completes the proof that, even if uninformed traders can obtain imprecise signals of information, feedback trading does not affect informed trader’s strategy nor the market price process.

Q.E.D.