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COMPETITIVE BUNDLING IN INFORMATION MARKETS: A SELLER-SIDE ANALYSIS

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Appendix A

Proofs for Lemmas and Propositions

Proof for Lemma 1

Substitution of Equation (2) in Equation (3) and algebraic rearrangement gives the result.

Proof for Lemma 2

Case A: Neither Seller Bundles Products

We derive prices by first deriving the seller reaction functions, $R_{i1}(p_{i2})$ and $R_{i2}(p_{i1})$, for sellers 1 and 2, respectively, when the products in the i^{th} competing pair are substitutes and when they are complements. The intersection of these reaction curves represents the Nash equilibrium.

(i) *Products are substitutes* (*i.e.,* $\bar{v}_i \leq 2v_i$).

We consider only the feasible and non-dominated region for p_{ij}, which is given by $(\bar{v}_i - v_i, v_i)$.

If one seller charges more than $\bar{v}_i - v_i$, then the other seller's optimal price is to charge slightly less than the first seller's price for the following reason: Suppose the first seller charges $p_{i1} > \bar{v}_i - v_i$. If the second seller charges more than p_{i1} (e.g., $p_{i1} + \delta$), the consumer's surplus or net utility from buying the product (i) only from firm 1, (ii) only from firm 2, and (iii) from both firms, are as follows:

(a) $v_i - p_{i1}$ when the consumer buys from only seller 1
(b) $v_i - p_{i1} - \delta$ when the consumer buys from only seller 2
(c) $\bar{v}_i - 2p_{i1} - \delta < 0$ when the consumer buys from both seller 1 and seller 2

Clearly, the consumer realizes maximum surplus by buying from only seller 1 and hence profit to seller 2 is zero.

If the second seller charges p_{i1}, the consumer's surplus or net utility from buying the product (a) only from firm 1, (b) only from firm 2, and (c) from both firms, are as follows:

(a) $v_i - p_{i1}$ when the consumer buys from only seller 1
(b) $v_i - p_{i1} + \delta$ when the consumer buys from only seller 2
(c) $\overline{v}_i - 2p_{i1} + \delta < 0$ when the consumer buys from both seller 1 and seller 2

Clearly, the consumer is indifferent between buying from only seller 1 and buying only from seller 2. Assuming that the consumer chooses the two sellers with equal probability, profit to seller 2 is $p_{i1}/2$.

If the second seller charges less than p_{i1} (e.g., $p_{i1} - \delta$), the consumer's surplus or net utility from buying the product (a) only from firm 1, (b) only from firm 2, and (c) from both firms, are as follows:

(a) $v_i - p_{i1}$ when the consumer buys from only seller 1
(b) $v_i - p_{i1} + \delta$ when the consumer buys from only seller 2
(c) $\overline{v}_i - 2p_{i1} + \delta < 0$ when the consumer buys from both seller 1 and seller 2

Clearly, the consumer realizes maximum surplus by buying from only seller 2 and hence profit to seller 2 is $p_{i1} - \delta$.

For any p_{i1}, there always exists a $\delta < p_{i1}/2$ such that setting a price $p_{i1} - \delta$ is optimal for seller 2.

Suppose one seller charges $\overline{v}_i - v_i$, then the other seller will charge $\overline{v}_i - v_i$ for the same reason explained above. Thus, we obtain the following reaction functions:

$$R_{i1}(p_{i2}) = \begin{cases} \overline{v}_i - v_i, if \;\; p_{i2} = \overline{v}_i - v_i \\ p_{2i} - \delta, if \;\; p_{i2} > \overline{v}_i - v_i \end{cases} \qquad R_{i2}(p_{i1}) = \begin{cases} \overline{v}_i - v_i, if \;\; p_{i1} = \overline{v}_i - v_i \\ p_{1i} - \delta, if \;\; p_{i1} > \overline{v}_i - v_i \end{cases}$$

The reaction curves intersect at $p_{i1} = p_{i2} = \overline{v}_i - v_i$.

(ii) *Products are complements* (*i.e.,* $\overline{v}_i > v_i$).

The feasible and non-dominated region for p_{i1} is given by $(v_i, \overline{v}_i - v_i)$. In this region, $R_{ij}(p_{i\bar{j}}) = \overline{v}_i - p_{i\bar{j}}, \forall j$, where $\bar{j} = 2$ if $j = 1$. The reaction curves intersect between the points $(\overline{v}_i - v_i, v_i)$ and $(v_i, \overline{v}_i - v_i)$. The equation of the intersection line is given by $p_{i1} + p_{i2} = \overline{v}_i$, and the set of points in this line constitute the Nash equilibria.

In the Nash Bargaining Solution, p_{i1} maximizes $(p_{i1} - v_i)(\overline{v}_i - p_{i1} - v_i)$ and p_{i2} maximizes $(p_{i2} - v_i)(\overline{v}_i - p_{i2} - v_i)$. Solving the above optimization problems simultaneously gives the unique equilibrium stated in the proposition.

Case B: Both Sellers Bundle Products

The proof for equilibrium bundle prices is similar to that for **Case A** after substituting the values for bundles in place of values for the i^{th} product.

Case C: One Seller Does Not Bundle and the Other Seller Bundles

Assume seller 1 bundles and seller 2 does not bundle. We consider three subcases: (i) products in both competing pairs are substitutes, (ii) products in both competing pairs are complements, and (iii) products in one pair are substitutes and the products in the other pair are complements.

(i) Products in both competing pairs are substitutes:

For seller 2, the feasible and non-dominated region for p_{i2} is given by $(\overline{v}_i - v_i, v_i)$. For seller 1, the feasible and non-dominated region for p_{i1} is given by $\left(\sum_{i=1}^{2}(\overline{v}_i - v_i), \sum_{i=1}^{2} v_i\right)$. The following reaction functions can be derived by using reasoning similar to that for **Case A**.

$$R_{b1}(p_{12},p_{22})=\begin{cases}\sum_{i=1}^{2}(\bar{v}_i-v_i), if\ p_{i2}=\bar{v}_i-v_i,\forall i\\ \sum_{i=1}^{2}p_{i2}-\delta, if\ \sum_{i-1}^{2}p_{i2}>\sum_{i=1}^{2}(\bar{v}_i-v_i)\end{cases}$$

$$\sum_{i=1}^{2}R_{i2}(p_{b1})=(\bar{v}_i-v_i)\ if\ p_{b1}=\sum_{i=1}^{2}(\bar{v}_i-v_i),\forall i$$

$$\sum_{i=1}^{2}R_{i2}(p_{b1})=p_{b1}-\delta\ if\ p_{b1}>\sum_{i=1}^{2}(\bar{v}_i-v_i)$$

Note that in the last equation of the above reaction functions, when seller 1 charges a price higher than $\sum_{i=1}^{2}(\bar{v}_i-v_i)$, seller 2 can sell both its products by setting a total price which is slightly below the bundle price set by seller 1.

The reaction curves intersect at $p_{b1}=\sum_{i=1}^{2}(\bar{v}_i-v_i), p_{i2}=(\bar{v}_i-v_i)$.

(ii) Products in both competing pairs are complements: The feasible and non-dominated regions are given by $(v_i,(\bar{v}_i-v_i))$ and $\left(\sum_{i=1}^{2}v_i,\sum_{i=1}^{2}(\bar{v}_i-v_i)\right)$ for p_{i2} and p_{b1}, respectively. In this region, $R_{b1}(p_{12},p_{22})=\sum_{i=1}^{2}(\bar{v}_i-v_i)-\sum_{i=1}^{2}p_{i2}$ and $\sum_{i=1}^{2}R_{i2}(p_{b1})=\sum_{i=1}^{2}(\bar{v}_i-v_i)-p_{b1}$. An analysis similar to that for **Case A** gives the unique symmetric equilibrium $p_{b1}=\sum_{i=1}^{2}p_{i2}=\sum_{i=1}^{2}\frac{\bar{v}_i}{2}$.

(iii) Products in one pair are substitutes and products in the other pair are complements.

We assume, without loss of generality, that products in pair 1 are substitutes and the products in pair 2 are complements. The feasible and non-dominated regions for p_{12} and p_{22}, respectively, are given by $((\bar{v}_i-v_i),v_i)$ and $(v_2,(\bar{v}_2-v_2))$. The feasible and non-dominated region for p_{b1} is given by $((\bar{v}_1-v_1)+v_2,v_1+(\bar{v}_2-v_2))$. In equilibrium, seller 1 will set a price for the bundle such that the bundle is sold. Consequently, the maximum value the buyer derives from product 1 from seller 2 is $(\bar{v}_1-v_1)$. Thus, $p_{12}=(\bar{v}_1-v_1)$, and the consumer will buy product 1 from seller 2 regardless of the price of the bundle. Consequently, given p_{22}, $p_{b1}=(\bar{v}_1-v_1)+\bar{v}_2=p_{22}$, and, given p_{b1}, $p_{22}=\bar{v}_2+\bar{v}_1-v_1-p_{b1}$. The unique equilibrium under NBS on this line is given by $p_{b1}-(\bar{v}_1-v_1)=p_{22}=\frac{\bar{v}_2}{2}$.

Proof for Proposition 1

We consider the following three cases: (i) when products in each competing pair are substitutes, (ii) when products in each pair are complements, and (iii) when products in the first pair are substitutes and the products in the second pair are complements. In each of these cases, we show the product policy game in strategic form and then derive the equilibrium strategy.

(i) When products in each pair are substitutes, the payoff table is given by the following:

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\sum_{i=1}^{2}\overline{v_i}-v_i,\sum_{i=1}^{2}\overline{v_i}-v_i\right)$	$\left(\sum_{i=1}^{2}(\overline{v_i}-v_i),\sum_{i=1}^{2}(\overline{v_i}-v_i)-\varepsilon\right)$
Bundle	$\left(\sum_{i=1}^{2}(\overline{v_i}-v_i)-\varepsilon,\sum_{i=1}^{2}(\overline{v_i}-v_i)\right)$	$\left(\sum_{i=1}^{2}(\overline{v_i}-v_i)-\varepsilon,\sum_{i=1}^{2}(\overline{v_i}-v_i)-\varepsilon\right)$

It is easy to see from the payoff table that the Nash equilibrium is (do-not-bundle, do-not-bundle).

(ii) When products in each pair are complements, the payoff table is as shown below.

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\sum_{i=1}^{2}\frac{\overline{v_i}}{2},\sum_{i=1}^{2}\frac{\overline{v_i}}{2}\right)$	$\left(\sum_{i=1}^{2}\frac{\overline{v_i}}{2},\sum_{i=1}^{2}\frac{\overline{v_i}}{2}-\varepsilon\right)$
Bundle	$\left(\sum_{i=1}^{2}\frac{\overline{v_i}}{2}-\varepsilon,\sum_{i=1}^{2}\frac{\overline{v_i}}{2}\right)$	$\left(\sum_{i=1}^{2}\frac{\overline{v_i}}{2}-\varepsilon,\sum_{i=1}^{2}\frac{\overline{v_i}}{2}-\varepsilon\right)$

It clearly follows that the Nash equilibrium is (do-not-bundle, do-not-bundle).

(iii) When products in the first pair are substitutes, and the products in the second pair are complements, we obtain the following payoff table.

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\overline{v_1}-v_1+\frac{\overline{v_2}}{2},\overline{v_1}-v_1+\frac{\overline{v_2}}{2}\right)$	$\left(\overline{v_1}-v_1+\frac{\overline{v_2}}{2},\overline{v_1}-v_1+\frac{\overline{v_2}}{2}-\varepsilon\right)$
Bundle	$\left(\overline{v_1}-v_1+\frac{\overline{v_2}}{2}-\varepsilon,\overline{v_1}-v_1+\frac{\overline{v_2}}{2}\right)$	$\begin{cases}\left(\overline{v_1}-v_1+\overline{v_2}-v_2-\varepsilon,\overline{v_1}-v_1+\overline{v_2}-v_2-\varepsilon\right) & \text{if bundles are substitutes}\\ \left(\frac{\overline{v_1}+\overline{v_2}}{2}-\varepsilon,\frac{\overline{v_1}+\overline{v_2}}{2}-\varepsilon\right) & \text{if bundles are complements}\end{cases}$

Because $\bar{v}_2-v_2>\frac{\bar{v}_2}{2}$ and $\bar{v}_1-v_1<\frac{\bar{v}_1}{2}$ (bundle, bundle) and (do-not-bundle, do-not-bundle) are both Nash equilibria regardless of whether the bundles behave as complements or as substitutes. Comparing the profits under these two equilibria shows that (bundle, bundle) is Pareto-dominant.

Proof for Proposition 2

Assume product 1 is a substitute and product 2 is a complement. In the bundling equilibrium, the consumer buys bundles from both sellers. Therefore, using the bundle price given in Lemma 2, we compute the consumer surplus and social welfare as follows:

Consumer surplus in the bundling equilibrium = $\begin{cases}2(v_1+v_2)-\overline{v_1}-\overline{v_2} & \text{if the bundles are substitutes}\\ 0 & \text{if the bundles are complements}\end{cases}$

Social welfare in the bundling equilibrium = $\overline{v_1}+\overline{v_2}$

In the unbundling scenario, the consumer still buys both products from each seller. Using the unbundled prices given in Lemma 2, we compute the consumer surplus and social welfare as the following:

Consumer surplus in the unbundling equilibrium = $2v_1-\overline{v_1}$

Social welfare in the unbundling equilibrium = $\overline{v_1}+\overline{v_2}$

Clearly, the consumer surplus is smaller under the bundling equilibrium than under unbundling because $2v_2-\overline{v_2}<0$. Furthermore, there is no difference in the social welfare in the two scenarios.

Proof for Proposition 3

Without loss of generality, we show that either pure bundling or unbundling weakly dominates mixed bundling for seller 1, regardless of seller 2's strategy.

Suppose seller 1 uses mixed bundling. We make the following observations.

(a) If the consumer buys the bundle from seller 1, then she will not buy either product 1 or product 2 individually from seller 1 at any positive price because the incremental value from either of these two individual products is zero.

(b) If the consumer does not buy the bundle from seller 1, then she will buy both products individually from seller 1 in the equilibrium because if a product is not bought then seller 1 can strictly increase its profit by setting a small price of $\varepsilon > 0$ and inducing the consumer to buy.

Now consider case (a). The equilibrium prices in the pricing subgame case are identical to

(i) those given in Lemma 2(b) if seller 2 sells only the pure bundle – both sellers obtain the same equilibrium profit as that when both sellers are pure bundlers.

(ii) those given in Lemma 2(c) if seller 2 sells only individual products—seller 2 obtains the same equilibrium profit as that when both sellers are pure unbundlers but seller 1 obtains less than the equilibrium profit of pure unbundlers because of the bundling cost.

(iii) those given in Lemma 2(b) if seller 2 also uses the mixed bundling strategy and it is profitable for seller 2 to sell the bundle than the individual products (i.e., when one product pair is a substitute and the other pair is a complement)—both sellers obtain the same equilibrium profit as that when both sellers are pure bundlers.

(iv) those given on Lemma 2(c) if seller 2 also uses the mixed bundling strategy and it is profitable for seller 2 to sell the individual products than the bundle (i.e., when both product pairs are either substitutes or complements)—both sellers obtain less than the equilibrium profit of pure unbundlers because of the bundling cost.

Clearly, in case (a), regardless of seller 2's strategy (i) – (iv), seller 1's prices and profit are never higher than to those when it uses pure bundling or pure unbundling.

Now consider case (b). The equilibrium prices in the pricing sub game case are identical to

(i) those given in Lemma 2(c) if seller 2 sells only the pure bundle—both sellers obtain less than the equilibrium profit of pure unbundlers because of the bundling cost.

(ii) those given in Lemma 2(a) if seller 2 sells only individual products—seller 2 obtains the same equilibrium profit as that when both sellers are pure unbundlers but seller 1 obtains less than the equilibrium profit of pure unbundlers because of the bundling cost.

(iii) those given in Lemma 2(c) if seller 2 also uses the mixed bundling strategy and it is profitable for seller 2 to sell the bundle rather than the individual products (i.e., when one product pair is a substitute and the other pair is a complement)—both sellers obtain less than the equilibrium profit of pure unbundlers because of the bundling cost.

(iv) those given on Lemma 2(a) if seller 2 also uses the mixed bundling strategy and the bundle price is not higher than sum of the prices of individual products for seller 2 (i.e., when both product pairs are either substitutes or complements)—both sellers obtain less than the equlibrium profit of pure unbundlers because of the bundling cost.

Clearly, in case (b), as in case (a), regardless of seller 2's strategy (i) – (iv), seller 1's prices and profit are never higher than those when it uses pure bundling or pure unbundling.

Proof for Lemma 3

Assume without loss of generality $v_{i1} > v_{i2}$. We prove Lemma 3 by deriving the seller reaction functions, $R_{i1}(p_{i2})$ and $R_{i2}(p_{i1})$, for sellers 1 and 2, respectively, under various conditions. The intersection of these reaction curves represents the Nash equilibrium.

Case (a)

(i) $\bar{v}_i \le v_{i1} + v_{i2}$

$$R_{i1} = \begin{cases} \overline{v_i} - v_{i2} \text{ if } p_{i2} \le \overline{v_i} - v_{i1} \\ v_{i1} - v_{i2} + p_{i2} - \varepsilon, \varepsilon > 0, \text{ if } \overline{v_i} - v_{i1} < p_{i2} \le v_{i2} \\ v_{i1} \text{ if } p_{i2} > v_{i2} \end{cases}$$

$$R_{i2} = \begin{cases} \overline{v_i} - v_{i1} \text{ if } p_{i1} < \overline{v_i} - v_{i2} \\ v_{i2} - v_{i1} + p_{i1} - \varepsilon, \varepsilon > 0, \text{ if } \overline{v_i} - v_{i1} < p_{i1} < v_{i1} \\ v_{i2} \text{ if } p_{i1} > v_{i1} \end{cases}$$

Hence, the Nash equilibrium prices are $\overline{v_i} - v_{i2}$ and $\overline{v_i} - v_{i1}$ for sellers 1 and 2 respectively.

(ii) $\overline{v}_i > v_{i1} + v_{i2}$

$$R_{i1} = \begin{cases} \overline{v_i} - v_{i2} \text{ if } p_{i2} \le v_{i2} \\ \overline{v_i} - p_{i2} \text{ if } v_{i2} < p_{i2} \le \overline{v_i} - v_{i1} \\ v_{i1} \text{ if } p_{i2} > \overline{v_i} - v_{i1} \end{cases}$$

$$R_{i2} = \begin{cases} \overline{v_i} - v_{i1} \text{ if } p_{i1} \le v_{i1} \\ \overline{v_i} - p_{i1} \text{ if } v_{i1} < p_{i1} \le \overline{v_i} - v_{i2} \\ v_{i2} \text{ if } p_{i1} > \overline{v_i} - v_{i2} \end{cases}$$

The reaction curves intersect between the points $\left(\overline{v}_i - v_{i2}, v_{i2}\right)$ and $\left(v_{i1}, \overline{v}_i - v_{i1}\right)$. The equation of the intersection line is given by $p_{i1} + p_{i2} = \overline{v}_i$, and the set of points in this line constitute the Nash equilibria. In the NBS solution, p_{i1} maximizes $\left(p_{i1} - v_{i1}\right)\left(\overline{v}_i - p_{i1} - v_{i2}\right)$ and p_{i2} maximizes $\left(p_{i2} - v_{i2}\right)\left(\overline{v}_i - p_{i2} - v_{i1}\right)$. Solving the above optimization problem gives the unique equilibrium stated in the Lemma.

Case (b)

The proof for Lemma 3 (b) is similar to that of Lemma 2 (b) after substitution of the reservation price values for bundles in place of reservation price values for the i^{th} product.

Case (c)

The feasible and non-dominated region for p_{i2} is given by $\left(\left(\overline{v_i} - v_{i1}\right), v_{i2}\right), \forall i \in \{t+1,...,n\}$ and $\left(v_{i2}, \left(\overline{v_i} - v_{i1}\right)\right), \forall i \in \{1,...,t\}$. The feasible and non-dominated region for p_{b1} is given by $\left(\sum_{i=t+1}^{n}\left(\overline{v_i} - v_{i1}\right) + \sum_{i=1}^{t} v_{i1}, \sum_{i=t+1}^{n} v_{i1} + \sum_{i=1}^{t}\left(\overline{v_i} - v_{i1}\right)\right)$.

In equilibrium, the consumer will buy the bundle and all products from the non-bundler. Otherwise, the sellers can increase their profit by setting the price of the product not bought to the incremental value it provides in which case the consumer will buy it.

Therefore, $p_{i2} = \overline{v_i} - v_{i1} \forall i \in \{t+1,...,n\}$, and the consumer will buy products $t + 1$ through n from seller 2 irrespective of the price of the bundle. Consequently, given $p_{i2}, i \in \{t+1,...,n\}$, $p_{b1} = \sum_{i=t+1}^{n}\left(\overline{v_i} - v_{i2}\right) + \sum_{i=1}^{t}\overline{v_i} - \sum_{i=1}^{t} p_{i2}$, and given p_{b1}, $\sum_{i=1}^{t} p_{i2} = \sum_{i=t+1}^{n}\left(\overline{v_i} - v_{i2}\right) + \sum_{i=1}^{t}\overline{v_i} - p_{b1}$.

The equilibrium under NBS is given by $p_{b1} = \sum_{i=t+1}^{n}\left(\overline{v_i} - v_{i2}\right) + \frac{\sum_{i=1}^{t}\overline{v_i}}{2}$ and $\sum_{i=1}^{t} p_{i2} = \sum_{i=1}^{t}\left(\frac{\overline{v_i}}{2} + \frac{v_{i2} - v_{i1}}{2}\right)$.

Proof for Proposition 4

We consider the following three cases: (i) when products in all competing pairs are substitutes, (ii) when products in all pairs are complements, and (iii) when products in the first t pairs are complements and the products in the rest of the pairs are substitutes. In each of these cases, we show the product policy game in strategic form and then derive the equilibrium strategy.

(i) When products in all pairs are substitutes, the payoff table is given by the following:

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\sum_{i=1}^{n}\left(\overline{v_i}-v_{i2}\right),\sum_{i=1}^{n}\left(\overline{v_i}-v_{i1}\right)\right)$	$\left(\sum_{i=1}^{n}\left(\overline{v_i}-v_{i2}\right),\sum_{i=1}^{n}\left(\overline{v_i}-v_{i1}\right)-\varepsilon\right)$
Bundle	$\left(\sum_{i=1}^{n}\left(\overline{v_i}-v_{i2}\right)-\varepsilon,\sum_{i=1}^{n}\left(\overline{v_i}-v_{i1}\right)\right)$	$\left(\sum_{i=1}^{n}\left(\overline{v_i}-v_{i2}\right)-\varepsilon,\sum_{i=1}^{n}\left(\overline{v_i}-v_{i1}\right)-\varepsilon\right)$

It is easy to see from the payoff table that the Nash equilibrium is (do-not-bundle, do-not-bundle).

(ii) When products in all pairs are complements, the payoff table is as shown below.

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i1}-v_{i2}}{2}\right),\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i2}-v_{i1}}{2}\right)\right)$	$\left(\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i1}-v_{i2}}{2}\right),\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i2}-v_{i1}}{2}\right)-\varepsilon\right)$
Bundle	$\left(\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i1}-v_{i2}}{2}\right)-\varepsilon,\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i2}-v_{i1}}{2}\right)\right)$	$\left(\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i1}-v_{i2}}{2}\right)-\varepsilon,\sum_{i=1}^{n}\left(\frac{\overline{v_i}}{2}+\frac{v_{i2}-v_{i1}}{2}\right)-\varepsilon\right)$

As for case (i), it clearly follows that the Nash equilibrium is (do-not-bundle, do-not-bundle).

(iii) When products in the first t pairs are complements, and the products in the other pairs are substitutes, we obtain the following payoff table:

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i2}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i1} - v_{i2}}{2}\right), \\ \sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i1}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i2} - v_{i1}}{2}\right)\end{array}\right)$	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i2}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i1} - v_{i2}}{2}\right), \\ \sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i1}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i2} - v_{i1}}{2}\right) - \varepsilon\end{array}\right)$
Bundle	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i2}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i1} - v_{i2}}{2}\right) - \varepsilon, \\ \sum_{i=t+1}^{n}\left(\bar{v}_i - v_{i2}\right) + \sum_{i=1}^{t}\left(\frac{\bar{v}_i}{2} + \frac{v_{i1} - v_{i2}}{2}\right)\end{array}\right)$	$\begin{cases}\left(\begin{array}{l}\sum_{i=1}^{n}\left(\bar{v}_i - v_{i2}\right) - \varepsilon, \\ \sum_{i=1}^{n}\left(\bar{v}_i - v_{i1}\right) - \varepsilon\end{array}\right) & \text{if bundles substitute} \\ \left(\begin{array}{l}\sum_{i=1}^{n}\left(\frac{\bar{v}_i}{2} + \frac{v_{i1} - v_{i2}}{2}\right) - \varepsilon, \\ \sum_{i=1}^{n}\left(\frac{\bar{v}_i}{2} + \frac{v_{i2} - v_{i1}}{2}\right) - \varepsilon\end{array}\right) & \text{if bundles complement}\end{cases}$

A comparison of the payoffs in the above table shows that (do-not-bundle, do-not-bundle) as well as (bundle, bundle) is an equilibrium. Comparing the profits under these two equilibria shows that (bundle, bundle) is Pareto-dominant.

Proof for Proposition 5

Proof: Note that the first t competing pair are complements and the others are substitutes. The following table shows the profit matrix for sellers.

	Do-not-bundle	Bundle
Do-not-bundle	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9}, \\ \sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9}\end{array}\right)$	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9}, \\ \sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9} - \varepsilon\end{array}\right)$
Bundle	$\left(\begin{array}{l}\sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9} - \varepsilon, \\ \sum_{i=t+1}^{n}\frac{(3v_i - \bar{v}_i)(\bar{v}_i - v_i)}{4v_i} + \sum_{i=1}^{t}\frac{\bar{v}_i}{9}\end{array}\right)$	$\begin{cases}\left(\frac{n\sum_{i=1}^{n}\left(\bar{v}_i - v_i\right)}{2} - \varepsilon, \frac{n\sum_{i=1}^{n}\left(\bar{v}_i - v_i\right)}{2} - \varepsilon\right) & \text{if bundles substitute} \\ \left(\frac{n\sum_{i=1}^{n}\bar{v}_i}{4} - \varepsilon, \frac{n\sum_{i=1}^{n}\bar{v}_i}{4} - \varepsilon\right) & \text{if bundles complement}\end{cases}$

A comparison of the payoffs in the above table yields the result given in the proposition.

Appendix B

Complementary and Substitution of Information for a Linear Predictive Model

Consider the following linear predictive model used by a decision maker to estimate quantity y.

$$y = \beta_1 x_1 + \beta_2 x_2$$

Assume the following prior distributions of predictor variables:

$$x_i \sim N(\overline{x_i}, \sigma_i^2)$$

where $N(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2. Further, let the correlation coefficient between x_1 and x_2 be ρ.

The quality s of estimate $\bar{y}$ to the decision maker is a linearly declining function of its variance (or mean squared error) given by the following:

$$x = 1 - \mathrm{var}(\bar{y})$$

Let the quality of estimate of y when variable i alone is obtained be denoted as s_i, the quality of estimate of y when both variables are obtained be denoted as s_{1+2}, and the quality of estimate of y when neither variable is obtained be denoted as s_0.

Further, denote the value of obtaining variable i alone as v_i and the values of obtaining both variables as v_{1+2}, and let the value of obtaining a variable (or both) be proportional to the improvement in quality of the estimate compared to when no variable is obtained. That is,

$$v_1 = ks_1 - ks_0,$$
$$v_2 = ks_2 - ks_0, \text{ and}$$
$$v_{1+2} = ks_{1+2} - ks_0$$

Using basic variance calculations, and using the fact that $\mathrm{var}(x_i|x_j) = (1 - \rho^2)\sigma_i^2$, we can compute the value of the estimate under different scenarios as the following:

$$v_1 = k(\beta_1^2\sigma_1^2 + \rho^2\beta_2^2\sigma_2^2 + 2\beta_1\beta_2\rho\sigma_1\sigma_2),$$
$$v_2 = k(\beta_2^2\sigma_2^2 + \rho^2\beta_1^2\sigma_1^2 + 2\beta_1\beta_2\rho\sigma_1\sigma_2), \text{ and}$$
$$v_{1+2} = k(\beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2 + 2\beta_1\beta_2\rho\sigma_1\sigma_2)$$

Comparing $v_1 + v_2$ with v_{1+2}, we find that $v_1 + v_2 < v_{1+2}$ if and only if $\dfrac{-2\beta_1\beta_2\sigma_1\sigma_2}{\beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2} < \rho < 0$.

Appendix C

Analysis of the Three-Product Case

There are two sellers, and each sells three products. Product i, $i \in \{1, 2, 3\}$ from each seller forms a competing pair. The two products in each competing pair are symmetric regarding their quality. The consumer utility from product i is v_i when i is bought from a single seller, and $\bar{v}_i$ when i from both sellers are bought. The products in a competing pair are complementors if $\bar{v}_i > 2v_i$ and substitutors if $\bar{v}_i \le 2v_i$. Hereafter, we refer to competing pair i as complementor (substitutor) pair if the products in that pair are complementors (substitutors). As in the two-product case, we assume that if a seller chooses to offer her products as a bundle, then she incurs a small bundling cost $\varepsilon > 0$. The bundling cost is the same regardless of the number of products bundled.

We have the following four possible scenarios depending on how many among the competing pairs are substitutors/complementors:
(a) All competing pairs are substitutors
(b) All competing pairs are complementors
(c) One pair is substitutor and the other two are complementors
(d) One pair is complementor and the other two are substitutors

Without loss of generality, we assume that competing pair 1 is the substitutor in scenario (c) and the complementor in scenario (d).

Regardless of the scenario, each seller has the following bundling/unbundling strategies in each:
A. Sell all products as a single bundle
B. Sell each product separately
C. Sell products 1 and 2 as a bundle, and sell product 3 separately
D. Sell products 1 and 3 as a bundle, and sell product 2 separately
E. Sell products 2 and 3 as a bundle, and sell product 1 separately

Now, we derive the sub game perfect equilibrium for each of the four scenarios (a) – (d). We denote the bundling strategies chosen by sellers in the first stage using the notation (x, y) where x denotes the strategy of the first seller, y denotes the strategy chosen by the second seller, and both x and y come from the set of strategies {A, B, C, D, E} discussed above.

In the following proof, we denote the bundle of all three products as (1 + 2 + 3), the bundle of product 1 and product 2 as (1 + 2), the bundle of product 2 and product 3 as (2 + 3), and the bundle of product 1 and product 3 as (1 + 3).

(a) All Competing Pairs Are Substitutors

Price equilibrium in second stage

1. Stage 1 strategy is (A, A). The competing bundles from the two sellers are substitutors (i.e., $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$. Using Proposition 3(b), we have price for each bundle as $\sum_{i=1}^{3}(\bar{v}_i - v_i)$.

2. Stage 1 strategy is (A, B). Using Proposition 3(c), we have $p_{(1+2+3)1} = \sum_{i=1}^{3}(\bar{v}_i - v_i), p_{i2} = (\bar{v}_i - v_i)$.

3. Stage 1 strategy is (A, C). The feasible and non-dominated regions for product prices result in the following constraints:

$$\sum_{i=1}^{3}(\bar{v}_i - v_i) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$
$$\bar{v}_3 - v_3 \le p_{32} \le v_3$$
$$\sum_{i=1}^{2}(\bar{v}_i - v_i) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$

In each of the above inequalities, the left hand side and the right hand side represent, respectively the minimum value and the maximum value the consumer obtains from the product (or bundle if two or more products are offered as a bundle). In this scenario, we further note that lower bound of any inequality also represents the maximum value realized by the consumer if she buys the competing product (or competing products of those in the bundle). The upper bound is the maximum value realized by the consumer if she does not buy the competing product (or competing products of those in the bundle).

In the equilibrium, both sellers will set prices so that all products are sold. This can be proved by contradiction. Assume that a product offered by a seller is not sold in the equilibrium. The seller of this product can profitably set the price of this product to the lower bound of the feasible region for that product, shown in the corresponding inequality above, and the consumer will purchase the product.

When every product is sold, the following constraints should be satisfied.

$$\sum_{i=1}^{3}\left(\overline{v_i}-v_i\right)\geq p_{(1+2+3)1}$$
$$\overline{v_3}-v_3\geq p_{32}$$
$$\sum_{i=1}^{2}\left(\overline{v_i}-v_i\right)\geq p_{(1+2)2}$$

Comparing the two sets of constraints for prices, we get the following unique equilibrium prices.

$$p_{(1+2+3)1}=\sum_{i=1}^{3}\left(\overline{v_i}-v_i\right),\ p_{(1+2)2}=\sum_{i=1}^{2}\left(\overline{v_i}-v_i\right),\ p_{32}=\left(\overline{v_3}-v_3\right)$$

4. Stage 1 strategy is (A, D). The proof is similar to the (A, C) case—the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in the (A, C) case. Applying the same logic as in the (A, C) case, we can show

$$p_{(1+2+3)1}=\sum_{i=1}^{3}\left(\overline{v_i}-v_i\right),\ p_{(1+3)2}=\left(\overline{v_1}-v_1\right)+\left(\overline{v_3}-v_3\right),\ p_{22}=\left(\overline{v_2}-v_2\right)$$

5. Stage 1 strategy is (A, E). Again, the proof is similar to the (A, C) case—the only difference is that seller 2 bundles products 2 and 3 in this case, whereas products 1 and 2 were bundled in the (A, C) case. Applying the same logic as in the (A, C) case, we can show

$$p_{(1+2+3)1}=\sum_{i=1}^{3}\left(\overline{v_i}-v_i\right),\ p_{(2+3)2}=\left(\overline{v_2}-v_2\right)+\left(\overline{v_3}-v_3\right),\ p_{12}=\left(\overline{v_1}-v_1\right)$$

6. Stage 1 strategy is (B, B). Using Proposition 3(a), we have each seller's price for product i as $\overline{v}_i-v_i$.

7. Stage 1 strategy is (B, C). Since product 3 is sold separately by each seller, product 3 from the two sellers forms a competing pair. Using Proposition 3(a), we have $p_{31}=p_{32}=\overline{v}_3-v_3$.

   For products 1 and 2, the case reduces to the two-product case in which both products are substitutes and, one sells the two products as a bundle and the other sells them separately. Using Proposition 1(c), we have the following result:

$$p_{i1}=\left(\overline{v_i}-v_i\right)\forall i\in\{1,2\},\ p_{(1+2)2}=\sum_{i=1}^{2}\left(\overline{v_i}-v_i\right)$$

8. Stage 1 strategy is (B, D). The proof is similar to the (B, C) case—the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in the (B, C) case. Applying the same logic as in the (B, C) case, we can show the following result:

$$p_{21}=p_{22}=\overline{v}_2-v_2\ \ p_{i1}=\left(\overline{v}_i-v_i\right)\forall i\in\{1,3\},p_{(1+3)2}=\left(\overline{v}_1-v_1\right)+\left(\overline{v}_3-v_3\right)$$

9. Stage 1 strategy is (B, E). Again, the proof is similar to the (B, C) case—the only difference is that seller 2 bundles product 2 with product 3 in this case, whereas it was product 2 that was bundled with product 1 in the (B, C) case. Applying the same logic as in the (B, C) case, we can show the following result:

$$p_{11} = p_{22} = \bar{v}_1 - v_1 \;\; p_{i1} = \left(\bar{v}_i - v_i\right) \forall i \in \{2, 3\}, p_{(1+3)2} = \left(\bar{v}_2 - v_2\right) + \left(\bar{v}_3 - v_3\right)$$

10. Stage 1 strategy is (C, C). Since product 3 is sold separately by each seller, product 3 from the two sellers forms a competing pair. Using Proposition 1(a), we have $p_{31} = p_{32} = \bar{v}_3 - v_3$.

    For the bundles with products 1 and 2, the case reduces to the two-product bundle case in which the bundles are substitutes and, both sellers bundle. Again, using Proposition 1(c), we have the following result:

$$p_{(1+2)2} = p_{1+2\_1} = \sum_{i=1}^{2} \left(\bar{v}_i - v_i\right)$$

11. Stage 1 strategy is (C, D). The feasible and non-dominated regions for product prices result in the following constraints.

$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le p_{(1+2)1} \le \sum_{i=1}^{2} v_i$$

$$\overline{v_3} - v_3 \le p_{31} \le v_3$$

$$\sum_{i \in \{1,3\}} \left(\overline{v_i} - v_i\right) \le p_{(1+3)2} \le \sum_{i \in \{1,3\}} v_i$$

$$\overline{v_2} - v_2 \le p_{22} \le v_2$$

    In each of the above inequalities, the left hand side and the right hand side represent, respectively the minimum value and the maximum value the consumer obtains from the product (or bundle if two or more products are offered as a bundle). In this scenario, we further note that lower bound of any inequality also represents the maximum value realized by the consumer if she buys the competing product (or competing products of those in the bundle). The upper bound is the maximum value realized by the consumer if she does not buy the competing product (or competing products of those in the bundle).

    In the equilibrium, both sellers will set prices so that all products are sold. This is can be proved by contradiction. Assume that a product offered by a seller is not sold in the equilibrium. The seller of this product can profitably set the price of this product to the lower bound of the feasible region for that product, shown in the corresponding inequality above, and the consumer will purchase the product.

    When every product is sold, the following constraints should be satisfied.

$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \ge p_{(1+2)1}$$

$$\overline{v_3} - v_3 \ge p_{31}$$

$$\sum_{i \in \{1,3\}} \left(\overline{v_i} - v_i\right) \ge p_{(1+3)2}$$

$$\overline{v_2} - v_2 \ge p_{22}$$

Comparing the two sets of constraints for prices, we get the following unique equilibrium prices.

$$p_{(1+2)1} = \sum_{i=1}^{2} \left(\overline{v_i} - v_i \right)$$
$$p_{31} = \overline{v_3} - v_3$$
$$p_{(1+3)2} = \sum_{i \in \{1,3\}} \left(\overline{v_i} - v_i \right)$$
$$p_{22} = \overline{v_2} - v_2$$

12. Stage 1 strategy is (C, E). This case is similar to (C, D). Applying the same logic as that for (C, D) we get the following equilibrium prices:

$$p_{(1+2)1} = \sum_{i=1}^{2} \left(\overline{v_i} - v_i \right)$$
$$p_{31} = \overline{v_3} - v_3$$
$$p_{(2+3)2} = \sum_{i=2}^{3} \left(\overline{v_i} - v_i \right)$$
$$p_{12} = \overline{v_1} - v_1$$

13. Stage 1 strategy is (D, D). This case is similar to (C, C). Applying the same logic as that for (C, C), we get $p_{21} = p_{22} = \overline{v}_2 - v_2$, $p_{(1+3)2} = p_{(1+3)1} = \left(\overline{v}_1 - v_1 \right) + \left(\overline{v}_3 - v_3 \right)$.

14. Stage 1 strategy is (D, E). Again, this case is similar to (C, D) except the products that are bundled are different in the two cases. Applying the same logic as that for (C, D) we get the following equilibrium prices:

$$p_{(1+3)1} = \sum_{i \in \{1,3\}} \left(\overline{v_i} - v_i \right)$$
$$p_{21} = \overline{v_3} - v_3$$
$$p_{(2+3)2} = \sum_{i=2}^{3} \left(\overline{v_i} - v_i \right)$$
$$p_{12} = \overline{v_1} - v_1$$

15. Stage 1 strategy pair is (E, E). This case is similar to (C, C). Applying the same logic as that for (C, C), we get $p_{11} = p_{12} = \overline{v}_1 - v_1$, $p_{(2+3)2} = p_{(2+3)1} = \left(\overline{v}_2 - v_2 \right) + \left(\overline{v}_3 - v_3 \right)$.

    The price equilibrium for other 10 possible strategy pairs can be determined from the above using symmetry.

Bundling equilibrium in the first stage

We have the following payoff matrix for the product policy game in the first stage. In each cell, the first expression is the payoff for seller 1 and the second expression is the payoff for seller 2.

Seller1/ Seller 2	A	B	C	D	E
A	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i), \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$
B	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i), \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)\end{pmatrix}$
C	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i), \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$
D	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i), \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$
E	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i), \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon, \\ \sum_{i=1}^{3}(\overline{v}_i-v_i)-\varepsilon\end{pmatrix}$

From the above payoff matrix, it is easy to see that (B,B) is the unique equilibrium.

(b) All Competing Pairs Are Complementors

Price equilibrium in second stage

1 Stage 1 strategy is (A, A)

The competing bundles from the two sellers are complementors, i.e., $\overline{v}_1+\overline{v}_2+\overline{v}_3 > 2(v_1+v_2+v_3)$

Using Proposition 3(b) , we have price for each bundle as $\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}$.

2. Stage 1 strategy is (A, B)

Using Proposition 3(c), we obtain $p_{(1+2+3)1} = \sum_{i=1}^{3} p_{i2} = \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}$

3. Stage 1 strategy is (A, C)

The feasible and non-dominated regions for product prices result in the following constraints.

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\overline{v_i} - v_i\right)$$

$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

$$\sum_{i=1}^{2} v_i \le p_{(1+2)2} \le \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right)$$

In each of the above inequalities, the left hand side and the right hand side represent, respectively the minimum value and the maximum value the consumer obtains from the product (or bundle if two or more products are offered as a bundle). In this scenario, we further note that lower bound of any inequality also represents the maximum value realized by the consumer if she buys only this product and not the competing product (or competing products of those in the bundle). The upper bound is the maximum value realized by the consumer if she also buys the competing product (or competing products of those in the bundle).

Since in the equilibrium, each seller will set prices so that every offering is sold, the following constraints should be satisfied.

$$p_{(1+2+3)1} + p_{32} + p_{(1+2)2} \le \sum_{i=1}^{3}\overline{v_i}$$

In the Nash Bargaining Solution, $p_{(1+2+3)1}$ maximizes $\left(p_{(1+2+3)1} - \sum_{i=1}^{3} v_t\right)\left(\sum_{i=1}^{3}\overline{v}_i - p_{(1+2+3)1} - \sum_{i=1}^{3} v_i\right)$ and $p_{32} + p_{(1+2)2}$ maximizes $\left(p_{32} + p_{1+2)2} - \sum_{i=1}^{3} v_i\right)\left(\sum_{i=1}^{3}\overline{v}_i - p_{32} - p_{(1+2)2} - \sum_{i=1}^{3} v_i\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{(1+2+3\_1} = p_{32} + p_{(1+2)2} = \frac{1}{2}\sum_{i=1}^{3}\overline{v}_i$.

4. Stage 1 strategy is (A, D)

The proof is similar to the (A, C) case; the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (A,C) case. Applying the same logic as in the (A,C) case, we can show the following:

$$p_{(1+2+3)1} = p_{22} + p_{(1+3)2} = \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}$$

5. Stage 1 strategy is (A, E)

Again, the proof is similar to the (A, C) case; the only difference is that seller 2 bundles products 2 and 3 in this case, whereas products 1 and 2 were bundled in (A,C) case. Applying the same logic as in the (A,C) case, we can show the following:

$$p_{(1+2+3)1} = p_{12} + p_{(2+3)2} = \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}$$

6. Stage 1 strategy is (B, B)

   Using Proposition 3 (a), we have each seller's price for product *i* as $\frac{\overline{v}_3}{2}$.

7. Stage 1 strategy is (B, C)

   Since product 3 is sold separately by each seller, product 3 from the two sellers form a competing pair. Since they are complements, using Proposition 1(a), we have $p_{31} = p_{32} = \frac{\overline{v}_3}{2}$.

   For products 1 and 2, the case reduces to the two-product case in which both products are complements and, one sells the two products as a bundle and the other sells them separately. Again, using Proposition 1(c), we have the following result:

$$\sum_{i=1}^{2} p_{i1} = p_{(1+2)2} = \frac{1}{2}\sum_{i=1}^{2}\overline{v_i}$$

8. Stage 1 strategy is (B, D)

   The proof is similar to the (B, C) case; the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (B,C) case. Applying the same logic as in the (B,C) case, we can show the following result:

$$p_{21} = p_{22} = \frac{\overline{v_2}}{2} \quad , \quad \sum_{i\in\{1,3\}} p_{i1} = p_{(1+3)2} = \frac{1}{2}\sum_{i\in\{1,3\}}\overline{v_i}$$

9. Stage 1 strategy is (B, E)

   Again, the proof is similar to the (B, C) case; the only difference is that seller 2 bundles product 2 with product 3 in this case, whereas it was product 2 that was bundled with product 1 in (B,C) case. Applying the same logic as in the (B,C) case, we can show the following result:

$$p_{11} = p_{12} = \frac{\overline{v_1}}{2} \quad , \quad \sum_{i\in\{2,3\}} p_{i1} = p_{(2+3)2} = \frac{1}{2}\sum_{i\in\{2,3\}}\overline{v_i}$$

10. Stage 1 strategy is (C, C)

    Since product 3 is sold separately by each seller, product 3 from the two sellers form a competing pair. Using Proposition 1(a), we have $p_{31} = p_{32} = \frac{\overline{v}_3}{2}$.

    For the bundles with products 1 and 2, the case reduces to the two-product bundle case in which the bundles are complementors and, both sellers bundle. Again, using Proposition 1(b), we have the following result: $p_{(1+2)2} = p_{(1\_2)1} = \frac{1}{2}\sum_{i=1}^{3}\overline{v}_i$.

11. Stage 1 strategy pair is (C, D)

    The feasible and non-dominated regions for product prices result in the following constraints.

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right)$$
$$v_3 \le p_{31} \le \overline{v_3} - v_3$$
$$\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\overline{v_i} - v_i\right)$$
$$v_2 \le p_{22} \le \overline{v_2} - v_2$$

In each of the above inequalities, the left hand side and the right hand side represent, respectively the minimum value and the maximum value the consumer obtains from the product (or bundle if two or more products are offered as a bundle). In this scenario, we further note that lower bound of any inequality also represents the maximum value realized by the consumer if she buys only this product and not the competing product (or competing products of those in the bundle). The upper bound is the maximum value realized by the consumer if she also buys the competing product (or competing products of those in the bundle).

In the equilibrium, both sellers will set prices so that all products are sold. This is can be proved by contradiction. Assume that a product offered by a seller is not sold in the equilibrium. The seller of this product can profitably set the price of this product to the lower bound of the feasible region for that product, shown in the corresponding inequality above, and the consumer will purchase the product.

When every product is sold, the following constraints should be satisfied.

$$p_{(1+2)1} + p_{31} + p_{22} + p_{(1+3)2} \le \sum_{i=1}^{3} \overline{v_i}$$

Using the Nash Bargaining Solution model discussed in the proof for the strategy (A, C), we obtain

$$p_{(1+2)1} + p_{31} = p_{22} + p_{(1+3)2} = \frac{1}{2}\sum_{i=1}^{3} \overline{v_i}$$

12. Stage 1 strategy is (C, E)

This case is similar to (C, D). Applying the same logic as that for (C,D) we get the following equilibrium prices.

$$p_{(1+2)1} + p_{31} = p_{12} + p_{(2+3)2} = \frac{1}{2}\sum_{i=1}^{3} \overline{v_i}$$

13. Stage 1 strategy is (D, D)

This case is similar to (C, C). Applying the same logic as that for (C, C), we get $p_{21} = p_{22} = \frac{\overline{v}_2}{2}, p_{(1+3)2} = p_{(1+3)1} = \frac{1}{2}\sum_{i\in\{1,3\}} \overline{v}_i$.

14. Stage 1 strategy is (D, E)

Again, this case is similar to (C, D) except the products that are bundled are different in the two cases. Applying the same logic as that for (C, D) we get the following equilibrium prices.

$$p_{(1+3)1} + p_{21} = p_{12} + p_{(2+3)2} = \frac{1}{2}\sum_{i=1}^{3} \overline{v_i}$$

15. Stage 1 strategy is (E, E)

    This case is similar to (C, C). Applying the same logic as that for (C, C), we get $p_{11} = p_{12} = \frac{\overline{v}_1}{2}, p_{(2+3)2} = p_{(2+3)1} = \frac{1}{2}\sum_{i=2}^{3}\overline{v}_i$.

    The price equilibrium for other 10 possible strategy pairs can be determined from the above using symmetry.

Bundling equilibrium in the first stage

We have the following payoff matrix for the product policy game in the first stage. In each cell, the first expression is the payoff for seller 1 and the second expression is the payoff for seller 2.

Seller1/ Seller 2	A	B	C	D	E
A	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$
B	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}\end{pmatrix}$
C	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$
D	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$
E	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}\left(\overline{v_i}-v_i\right), \\ \sum_{i=1}^{3}\left(\overline{v_i}-v_i\right)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon, \\ \frac{1}{2}\sum_{i=1}^{3}\overline{v_i}-\varepsilon\end{pmatrix}$

From the above payoff matrix, it is easy to see that (B,B) is the unique equilibrium.

(c) One Pair Is Substitutor and the Other Two Are Complementors

Without loss of generality, we assume that competing pair 1 is the substitutor and the other two pairs are complementors. That is, $\overline{v}_1 \le 2v_1, \overline{v}_2 > 2v_2$, and $\overline{v}_3 > 2v_3$.

Price equilibrium in second stage

1. Stage 1 strategy is (A, A)

   We need to consider two cases: (i) the competing bundles are substitutors (i.e., $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$), and (ii) the competing bundles are complementors (i.e., $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$).

   (i) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$

   Using Proposition 3 (b) for the case of competing substitutors, we have price for each bundle as $\sum_{i=1}^{3}(\overline{v}_i - v_i)$.

   (ii) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$

   Using Proposition 3 (b) for the case of competing complementors, we have price for each bundle as $\sum_{i=1}^{3}\frac{\overline{v}_i}{2}$.

2. Stage 1 strategy is (A, B)

The feasible and non-dominated regions for seller 1's product price depends on whether $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$, or $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$ is satisfied.

(i) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{3}\left(\overline{v_i} - v_i\right) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$
$$\overline{v_1} - v_1 \le p_{12} \le v_1$$
$$v_2 \le p_{22} \le \overline{v_2} - v_2$$
$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

In each of the above inequalities, the left hand side and the right hand side represent, respectively the minimum value and the maximum value the consumer obtains from the product (or bundle if two or more products are offered as a bundle). In the first and second inequalities, the lower bound of any inequality also represents the maximum value realized by the consumer if she buys the competing product (or competing products of those in the bundle), and the upper bound is the maximum value realized by the consumer if she does not buy the competing product (or competing products of those in the bundle). In the other inequalities, lower bound of the inequality also represents the maximum value realized by the consumer if she buys only this product and not the competing product (or competing products of those in the bundle), and the upper bound is the maximum value realized by the consumer if she also buys the competing product (or competing products of those in the bundle).

In the equilibrium, both sellers will set prices so that all products are sold. This is can be proved by contradiction. Assume that a product offered by a seller is not sold in the equilibrium. The seller of this product can profitably set the price of this product to the lower bound of the feasible region for that product, shown in the corresponding inequality above, and the consumer will purchase the product.

When every product is sold, for p_{12}, the following constraint should be satisfied: $\overline{v}_1 - v_1 = p_{12}$. Furthermore, for seller 1, the value of product 1 in the bundle will be equal to $\overline{v}_1 - v_1$. Therefore, the following constraint should be satisfied for $p_{(1+2+3)1}$:

$$\sum_{i=2}^{3} v_i + \overline{v_1} - v_1 \le p_{(1+2+3)1} \le \sum_{i=2}^{3}\left(\overline{v_i} - v_i\right) + \overline{v_1} - v_1$$

In the Nash Bargaining Solution, $p_{(1+2+3)1}$ maximizes $\left(p_{(1+2+3)1} - \left(\sum_{i=2}^{3} v_i + \bar{v}_1 - v_1\right)\right)\left(\sum_{i=2}^{3}\bar{v}_i + \bar{v}_1 - v_1 - p_{(1+2+3)1} - \sum_{i=2}^{3} v_i\right)$ and $p_{32} + p_{22}$ maximizes $\left(p_{32} + p_{22} - \sum_{i=2}^{3} v_i\right)\left(\sum_{i=2}^{3}\bar{v}_i + \bar{v}_1 - v_1 - p_{32} + p_{22} - \left(\sum_{i=2}^{3} v_i + \bar{v}_1 - v_i\right)\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{(1+2+3)1} - \bar{v}_1 + v_1 = p_{22} + p_{32} = \frac{1}{2}\sum_{i=2}^{3}\bar{v}_i$.

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\overline{v_i} - v_i\right)$$
$$\overline{v_1} - v_1 \le p_{12} \le v_1$$
$$v_2 \le p_{22} \le \overline{v_2} - v_2$$
$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

In the equilibrium, both sellers will set prices so that all products are sold.

When every product is sold, for p_{12}, the following constraints should be satisfied: $\bar{v}_1 - v_1 = p_{12}$. Furthermore, for seller 1, the value of product 1 in the bundle will be equal to $\bar{v}_1 - v_1$. Therefore, the following constraint should be satisfied for $p_{(1+2+3)1}$.

$$\sum_{i=2}^{3} v_i + \overline{v_1} - v_1 \le p_{(1+2+3)1} \le \sum_{i=2}^{3}\left(\overline{v_i} - v_i\right) + \overline{v_1} - v_1$$

In the Nash Bargaining Solution, $p_{(1+2+3}$ maximizes $\left(p_{(1+2+3)1} - \left(\sum_{i=2}^{3} v_i + \bar{v}_1 - v_1\right)\right)\left(\sum_{i=2}^{3}\bar{v}_i + \bar{v}_1 - v_1 - p_{1+2+3)1} - \sum_{i=2}^{3} v_i\right)$ and $p_{32} + p_{22}$ maximizes $\left(p_{32} + p_{22} - \sum_{i=2}^{3} v_i\right)\left(\sum_{i=2}^{3}\bar{v}_i + \bar{v}_1 - v_1 - p_{32} + p_{22} - \left(\sum_{i=2}^{3} v_i + \bar{v}_1 - v_1\right)\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{(1+2+3)1} - \bar{v}_1 + v_1 = p_{22} + p_{32} = \frac{1}{2}\sum_{i=2}^{3}\bar{v}_i$.

3. Stage 1 strategy is (A, C)

For seller 1, we need to consider two cases: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$. For seller 2, the feasible and non-dominated regions for p_{32} is given by $\left(v_3, \bar{v}_3 - v_3\right)$, and the feasible and non-dominated region for $p_{(1+2)2}$ depends on whether $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$ or $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$. So, the price equilibrium depends on which of the following conditions are satisfied:

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$
(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$
(iii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$

Note that $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ is not possible because $\bar{v}_1 \le 2v_1, \bar{v}_2 > 2v_2$, and $\bar{v}_3 > 2v_3$ in this scenario.

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$ and $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3}\left(\overline{v_i} - v_i\right) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$
$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$
$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) = p_{(1+2)2}$$
$$v_3 + \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le p_{(1+2+3)1} \le \overline{v_3} - v_3 + \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right)$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2+3)1} - \sum_{i=1}^{3}(\bar{v}_i - v_i) = p_{32} = \frac{\bar{v}_3}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$ and $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\overline{v_i} - v_i\right)$$
$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$
$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) = p_{(1+2)2}$$
$$v_3 \le p_{(1+2+3)1} - \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le \overline{v_3} - v_3$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2+3)1} - \sum_{i=1}^{3}(\bar{v}_i - v_i) = p_{32} = \frac{\bar{v}_3}{2}$.

(iii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$ and $\bar{v}_1 + \bar{v}_2 \ge 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\overline{v_i} - v_i\right)$$
$$\sum_{i=1}^{2} v_i \le p_{(1+2)2} \le \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right)$$
$$v_3 \le p_{32} \le \overline{v_3} - v_3$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2+3)1} = p_{(1+2)2} + p_{32} = \sum_{i=1}^{3}\frac{\overline{v}_i}{2}$.

4. Stage 1 strategy is (A, D)

The proof is similar to the (A, C) case; the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (A, C) case. Applying the same logic as in the (A, C) case, we can show that there are three possible combinations of conditions to consider, and that the equilibrium prices can be shown to be the following:

(i) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$ and $\overline{v}_1 + \overline{v}_2 \le 2(v_1 + v_2)$

$$\sum_{i\in\{1,3\}}\left(\overline{v_i} - v_i\right) = p_{(1+3)2}$$
$$p_{(1+2+3)1} - \sum_{i\in\{1,3\}}\left(\overline{v_i} - v_i\right) = p_{22} = \frac{\overline{v_2}}{2}$$

(ii) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$ and $\overline{v}_1 + \overline{v}_2 \le 2(v_1 + v_2)$

$$\sum_{i\in\{1,3\}}\left(\overline{v_i} - v_i\right) = p_{(1+3)2}$$
$$p_{(1+2+3)1} - \sum_{i\in\{1,3\}}\left(\overline{v_i} - v_i\right) = p_{22} = \frac{\overline{v_2}}{2}$$

(iii) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$ and $\overline{v}_1 + \overline{v}_2 \ge 2(v_1 + v_2)$

$$p_{(1+2+3)1} = p_{(1+3)2} + p_{22} = \sum_{i=1}^{3}\frac{\overline{v_i}}{2}$$

5. Stage 1 strategy is (A, E)

For seller 1, we need to consider two cases: (i) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$, and (ii) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$.
For seller 2, the feasible and non-dominated regions for p_{12} is given by $(\overline{v}_1 - v_1, v_1)$, and the feasible and non-dominated region for $p_{(2+3)2}$ is given by $\left(\sum_{i=2}^{3} v_i, \sum_{i=2}^{3}(\overline{v}_i - v_i)\right)$.

So, the price equilibrium depends on which of the following conditions are satisfied:

(i) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2(v_1 + v_2 + v_3)$
(ii) $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2(v_1 + v_2 + v_3)$.

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3}\left(\bar{v}_i - v_i\right) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$
$$\sum_{i=2}^{3} v_i \le p_{(2+3)2} \le \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right)$$
$$\bar{v}_1 - v_1 \le p_{12} \le v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\bar{v}_1 - v_1 = p_{12}$$
$$\sum_{i=2}^{3} v_i \le p_{(1+2+3)1} - \bar{v}_1 + v_1 \le \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right)$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2+3)1} - \bar{v}_1 + v_1 = \sum_{i-2}^{3} \frac{\bar{v}_i}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\bar{v} - v_i\right)$$
$$\sum_{i=2}^{3} v_i \le p_{(2+3)2} \le \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right)$$
$$\bar{v}_1 - v_1 \le p_{12} \le v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\bar{v}_1 - v_1 = p_{12}$$
$$\sum_{i=2}^{3} v_i \le p_{(1+2+3)1} - \bar{v}_1 + v_1 \le \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right)$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2+3)1} - \bar{v}_1 + v_1 = p_{(2+3)2} = \sum_{i-2}^{3} \frac{\bar{v}_i}{2}$.

6. Stage 1 strategy is (B, B)

Using Proposition 3 (a) , we have each seller's price for product 1 as $\bar{v}_1 - v_1$, for product 2 as $\frac{\bar{v}_2}{2}$, and for product 3 as $\frac{\bar{v}_3}{2}$.

7. Stage 1 strategy is (B, C)

For seller 1, the feasible and non-dominated region for p_{11} is given by $\left(\bar{v}_1 - v_1, v_1\right)$, that for p_{21} is given by $\left(v_2, \bar{v}_2 - v_2\right)$, and that for p_{32} is given by $\left(v_3, \bar{v}_3 - v_3\right)$. For seller 2, the feasible and non-dominated regions for p_{32} is given by $\left(v_3, \bar{v}_3 - v_3\right)$. The feasible and non-dominated regions for $p_{(1+2)2}$ depends on whether $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$ or $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$.

So, the price equilibrium depends on which of the following conditions are satisfied:

(i) $\bar{v}_1 + \bar{v}_2 \le 2\left(v_l + v_2\right)$

(ii) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$

(i) $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_l + v_2\right)$

The following should hold in the equilibrium:

$$\begin{aligned}
&\overline{v_1} - v_1 \le p_{11} \le v_1 \\
&v_2 \le p_{21} \le \overline{v_2} - v_2 \\
&v_3 \le p_{31} \le \overline{v_3} - v_3 \\
&\sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i \\
&v_3 \le p_{32} \le \overline{v_3} - v_3
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\begin{aligned}
&\overline{v_1} - v_1 = p_{11} \\
&v_2 \le p_{(1+2)2} - \overline{v_1} + v_1 \le \overline{v_2} - v_2
\end{aligned}$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2)2} - \bar{v}_1 + v_1 = p_{21} = \frac{\bar{v}_2}{2}$ and $p_{32} = p_{31} = \frac{\bar{v}_3}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$

The following should hold in the equilibrium:

$$\begin{aligned}
&\overline{v_1} - v_1 \le p_{11} \le v_1 \\
&v_2 \le p_{21} \le \overline{v_2} - v_2 \\
&v_3 \le p_{31} \le \overline{v_3} - v_3 \\
&\sum_{i=1}^{2} v_i \le p_{(1+2)2} \le \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \\
&v_3 \le p_{32} \le \overline{v_3} - v_3
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\begin{aligned}
&\overline{v_1} - v_1 = p_{11} \\
&v_2 \le p_{(1+2)2} - \overline{v_1} + v_1 \le \overline{v_2} - v_2
\end{aligned}$$

Using the Nash Bargaining Solution model discussed for the strategy (A, B), we obtain $p_{(1+2)2} - \bar{v}_1 + v_1 = p_{21} = \frac{\bar{v}_2}{2}$ and $p_{32} = p_{31} = \frac{\bar{v}_3}{2}$.

8. Stage 1 strategy is (B, D)

The proof is similar to the (B, C) case – the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (B,C) case. Applying the same logic as in the (B,C) case, we can show the following result:

$$p_{11} = \overline{v_1} - v_1 \quad , \quad p_{(1+2)2} - \overline{v_1} + v_1 = p_{21} = \frac{\overline{v_2}}{2} \quad \text{and} \quad p_{32} = p_{31} = \frac{\overline{v_3}}{2}$$

9. Stage 1 strategy is (B, E)

For seller 1, the feasible and non-dominated region for p_{11} is given by $(\bar{v}_1 - v_1, v_1)$, that for p_{21} is given by $(v_2, \bar{v}_2 - v_2)$, and that for p_{31} is given by $(v_3, \bar{v}_3 - v_3)$. For seller 2, the feasible and non-dominated regions for p_{12} is given by $(\bar{v}_1 - v_1, v_1)$. The feasible and non-dominated regions for $p_{(2+3)2}$ is given by $\left(\sum_{i=2}^{3} v_i, \sum_{i=2}^{3}(\bar{v}_i - v_i)\right)$.

The following should hold in the equilibrium:

$$\begin{aligned}
&\overline{v_1} - v_1 \le p_{11} \le v_1 \\
&v_2 \le p_{21} \le \overline{v_2} - v_2 \\
&v_3 \le p_{31} \le \overline{v_3} - v_3 \\
&\overline{v_1} - v_1 \le p_{12} \le v_1 \\
&\sum_{i=2}^{3} v_i \le p_{(2+3)2} \le \sum_{i=2}^{3}\left(\overline{v_i} - v_i\right)
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\overline{v_1} - v_1 = p_{11} = p_{12}$$

Using the Nash Bargaining Solution model discussed for the strategy (A,B), we obtain $p_{(2+3)2} = p_{21} + p_{31} = \sum_{i=2}^{3} \frac{\bar{v}_i}{2}$.

10. Stage 1 strategy is (C, C)

Since product 3 is sold separately by each seller, product 3 from the two sellers form a competing pair. Since they are complements, using Proposition 1(a), we have $p_{31} = p_{32} = \frac{\bar{v}_3}{2}$.

For the bundles with products 1 and 2, the case reduces to the two-product bundle case. The bundles can be substitutes or complements. So, using Proposition 1(b), we have the following result:

$$p_{(1+2)2} = p_{(1+2)1} = \sum_{i=1}^{2}\left(\overline{v_i} - v_i\right) \text{ if } 2\sum_{i=1}^{2} v_i \le \sum_{i=1}^{2} \overline{v_i}$$

$$p_{(1+2)2} = p_{(1+2)1} = \frac{\sum_{i=1}^{2} \overline{v_i}}{2} \text{ if } 2\sum_{i=1}^{2} v_i > \sum_{i=1}^{2} \overline{v_i}$$

11. Stage 1 strategy is (C, D)

For seller 1, the feasible and non-dominated region for p_{31} is given by $\left(v_3, \bar{v}_3 - v_3\right)$. The feasible and non-dominated region for $p_{(1+2)1}$ depends on whether $2\sum_{i=1}^{2} v_i \leq \sum_{i=1}^{2} \bar{v}_i$ or $2\sum_{i=1}^{2} v_i > \sum_{i=1}^{2} \bar{v}_i$. For seller 2, the feasible and non-dominated region for p_{22} is given by $\left(v_2, \bar{v}_2 - v_2\right)$. The feasible and non-dominated region for $p_{(1+3)2}$ depends on whether $2\sum_{i\in\{1,3\}} v_i \leq \sum_{i\in\{1,3\}} \bar{v}_i$ or if $2\sum_{i\in\{1,3\}} v_i > \sum_{i\in\{1,3\}} \bar{v}_i$.

So, the price equilibrium depends on which of the following conditions are satisfied:

(i) $\bar{v}_1 + \bar{v}_2 \leq 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 \leq 2\left(v_1 + v_3\right)$

(ii) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 \leq 2\left(v_1 + v_3\right)$

(iii) $\bar{v}_1 + \bar{v}_2 \leq 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 > 2\left(v_1 + v_3\right)$

(iv) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 > 2\left(v_1 + v_3\right)$

(i) $\bar{v}_1 + \bar{v}_2 \leq 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 \leq 2\left(v_1 + v_3\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2}\left(\bar{v}_1 - v_1\right) \leq p_{(1+2)1} \leq \sum_{i=1}^{2} v_i$$
$$\sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right) \leq p_{(1+3)2} \leq \sum_{i\in\{1,3\}} v_i$$
$$v_3 \leq p_{31} \leq \bar{v}_3 - v_3$$
$$v_2 \leq p_{22} \leq \bar{v}_2 - v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) = p_{(1+2)1}$$
$$\sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right) = p_{(1+3)2}$$

Given the above prices for the bundles, $p_{31} + p_{22}$ cannot exceed $2v_1 + v_2 + v_3 - \bar{v}_1$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \leq 2\left(v_1 + v_2 + v_3\right)$, the constraints $v_3 \leq p_{31} \leq \bar{v}_3 - v_3$, $v_2 \leq p_{22} \leq \bar{v}_2 - v_2$ are more stringent than the non-negative surplus constraint on $p_{31} + p_{22}$. Therefore, $p_{31} = \bar{v}_3 - v_3, p_{22} = \bar{v}_2 - v_2$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$, we have $p_{31} + p_{22} = 2v_i + v_2 + v_3 - \bar{v}_1$. Therefore, in the Nash Bargaining Solution, p_{31} maximizes $\left(p_{31} - v_3\right)\left(2v_1 + v_2 + v_3 - \bar{v}_1 - p_{31} - v_2\right)$ and p_{22} maximizes $\left(p_{22} - v_2\right)\left(2v_1 + v_2 + v_3 - \bar{v}_1 - p_{22} - v_3\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{31} = v_1 + v_3 - \frac{\bar{v}_2}{2}$ and $p_{22} = v_1 + v_2 - \frac{\bar{v}_2}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 \leq 2\left(v_1 + v_3\right)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2} \left(\bar{v}_i - v_i\right)$$
$$\sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right) \le p_{(1+3)2} \le \sum_{i\in\{1,3\}} v_i$$
$$v_3 \le p_{31} \le \bar{v}_3 - v_3$$
$$v_2 \le p_{22} \le \bar{v}_2 - v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right)$$

Given the above price, $p_{31} + p_{22} + p_{(1+2)1}$ cannot exceed $\sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$, the constraints $v_3 \le p_{31} \le \bar{v}_3 - v_3, v_2 \le p_{22} \le \bar{v}_2 - v_2, \sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2} \left(\bar{v}_i - v_i\right)$ are more stringent than the non-negative surplus constraint on $p_{31} + p_{22} + p_{(1+2)1}$. Therefore, $p_{31} = \bar{v}_3 - v_3, p_{22} = \bar{v}_2 - v_2, p_{(1+2)1} = \sum_{i=1}^{2} \left(\bar{v}_i - v_i\right)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$, we have $p_{31} + p_{22} + p_{(1+2)1} = \sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right)$. Therefore, in the Nash Bargaining Solution, $p_{31} + p_{(1+2)1}$ maximizes $\left(p_{31} + p_{(1+2)1} - \sum_{i=1}^{3} v_i\right)\left(\sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right) - p_{31} - p_{(1+2)1} - v_2\right)$ and p_{22} maximizes $\left(p_{22} - v_2\right)\left(\sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right) - p_{22} - \sum_{i=1}^{3} v_i\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{31} + p_{(1+2)1} = v_1 + v_3 + \frac{\bar{v}_2}{2}$ and $p_{22} = \frac{\bar{v}_2}{2}$.

(iii) $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 > 2\left(v_1 + v_3\right)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{2} \left(\bar{v}_i - v_i\right) \le p_{(1+2)1} \sum_{i=1}^{2} v_i$$
$$\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}} \left(\bar{v}_i - v_i\right)$$
$$v_3 \le p_{31} \le \bar{v}_3 - v_3$$
$$v_2 \le p_{22} \le \bar{v}_2 - v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\sum_{i=1}^{2}\left(\overline{v}_i - v_i\right) = p_{(1+2)1}$$

Given the above price, $p_{31} + p_{22} + p_{(1+2)1}$ cannot exceed $\sum_{i=1}^{3}\overline{v}_i - \sum_{i=1}^{2}\left(\overline{v}_i - v_i\right)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$, the constraints $v_3 \le p_{31} \le \overline{v}_3 - v_3, v_2 \le p_{22} \le \overline{v}_2 - v_2$, $\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\overline{v}_i - v_i\right)$ are more stringent than the non-negative surplus constraint on $p_{31} + p_{22} + p_{(1+2)1}$. Therefore, $p_{31} = \overline{v}_3 - v_3, p_{22} = \overline{v}_2 - v_2, p_{(1+3)2} = \sum_{i\in\{1,3\}}\left(\overline{v}_i - v_i\right)$.

If $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 > 2\left(v_1 + v_2 + v_3\right)$, we have $p_{31} + p_{22} + p_{(1+3)2} = \sum_{i=1}^{3}\overline{v}_i - \sum_{i=1}^{2}\left(\overline{v}_i - v_i\right)$. Therefore, in the Nash Bargaining Solution, p_{31} maximizes $\left(p_{31} - v_3\right)\left(\sum_{i=1}^{3}\overline{v}_i - \sum_{i=1}^{2}\left(\overline{v}_i - v_i\right) - p_{31} - \sum_{i=1}^{3} v_i\right)$ and $p_{22} + p_{(1+2)1}$ maximizes $\left(p_{22} + p_{(1+3)2} - \sum_{i=1}^{3} v_i\right)\left(\sum_{i=1}^{3}\overline{v}_i - \sum_{i=1}^{2}\left(\overline{v}_i - v_i\right) - p_{22} - p_{(1+3)2} - v_3\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{31} = \frac{\overline{v}_3}{2}$ and $p_{22} + p_{(1+3)2} = \frac{\overline{v}_3 + 2v_1 + 2v_2}{2}$.

(iv) $\overline{v}_1 + \overline{v}_2 > 2\left(v_1 + v_2\right)$ and $\overline{v}_1 + \overline{v}_3 > 2\left(v_1 + v_3\right)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{2}\left(\overline{v}_1 - v_1\right) \le p_{(1+2)1} \le \sum_{i=1}^{2}\left(\overline{v}_i - v_i\right)$$
$$\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\overline{v}_i - v_i\right)$$
$$v_3 \le p_{31} \le \overline{v}_3 - v_3$$
$$v_2 \le p_{22} \le \overline{v}_2 - v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, and collectively every product offering (single or bundle) is a complement to the competing offerings, using Nash Bargaining Solution, the following characterizes the equilibrium prices:

$$p_{(1+2)1} + p_{31} = p_{(1+3)2} + p_{22} = \sum_{i=1}^{3}\frac{\overline{v}_i}{2}$$

12. Stage 1 strategy is (C, E)

For seller 1, the feasible and non-dominated region for p_{31} is given by $\left(v_3, \overline{v}_3 - v_3\right)$. The feasible and non-dominated region for $p_{(1+2)1}$ depends on whether $2\sum_{i=1}^{2} v_i \le \sum_{i=1}^{2}\overline{v}_i$ or $2\sum_{i=1}^{2} v_i > \sum_{i=1}^{2}\overline{v}_i$. For seller 2, the feasible and non-dominated region for p_{12} given by $\left(\overline{v}_1 - v_1, v_1\right)$. The feasible and non-dominated region for $p_{(2+3)2}$ is given by $\left(\sum_{i\in\{2,3\}} v_i, \sum_{i\in\{2,3\}}\left(\overline{v}_i - v_i\right)\right)$.

So, the price equilibrium depends on which of the following conditions are satisfied:

(i) $\overline{v}_1 + \overline{v}_2 \le 2\left(v_1 + v_2\right)$

(ii) $\overline{v}_1 + \overline{v}_2 > 2\left(v_1 + v_2\right)$

(i) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{2}(\bar{v}_i - v_i) \le p_{(1+2)1} \le \sum_{i=1}^{2} v_i$$
$$\sum_{i\in\{2,3\}} v_i \le p_{(2+3)2} \le \sum_{i\in\{2,3\}}(\bar{v}_i - v_i)$$
$$v_3 \le p_{31} \le \bar{v}_3 - v_3$$
$$v_1 - v_1 \le p_{12} \le v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\sum_{i=1}^{2}(\bar{v}_i - v_i) = p_{(1+2)1}$$
$$\bar{v}_1 - v_1 = p_{12}$$

Given the above prices, $p_{31} + p_{12} + p_{(2+3)2}$ cannot exceed $\sum_{i=1}^{3}\bar{v}_1 - \sum_{i=1}^{2}(\bar{v}_i - v_i) - (\bar{v}_1 - v_1)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, the constraints $\sum_{i\in\{2,3\}} v_i \le p_{(2+3)2} \le \sum_{i\in\{2,3\}}(\bar{v}_i - v_i) - (\bar{v}_1 - v_1)$ are more stringent than the non-negative surplus constraint on $p_{31} + p_{12} + p_{(2+3)2}$. Therefore, $p_{(2+3)2} = \sum_{i\in\{2,3\}}(\bar{v}_i - v_i), p_{31} = \bar{v}_3 - v_3$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, we have $p_{31} + p_{12} + p_{(2+3)2} = \sum_{i=1}^{3}\bar{v}_i - \sum_{i=1}^{3}(\bar{v}_i - v_i)(\bar{v}_1 - v_1)$. Therefore, in the Nash Bargaining Solution, p_{31} maximizes $(p_{31} - v_3)\left(\sum_{i=1}^{3}\bar{v}_i - \sum_{i=1}^{2}(\bar{v}_i - v_i) - p_{31} - \sum_{i=1}^{3} v_i\right)$ and $p_{12} + p_{(2+3)2}$ maximizes $\left(p_{12} + p_{(2+3)2} = \sum_{i=1}^{3} v_i\right)\left(\sum_{i=1}^{3}\bar{v}_i - \sum_{i=1}^{2}(\bar{v}_i - v_i) - (\bar{v}_1 - v_1) - p_{12} - p_{(2+3)2} - v_3\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{32} = \frac{\bar{v}_3 + v_1 - \bar{v}_1}{2}$ and $p_{12} + p_{(2+3)2} = \frac{\bar{v}_3 + 3v_2 - \bar{v}_1}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2}(\bar{v}_i - v_i)$$
$$\sum_{i\in\{2,3\}} v_i \le p_{(2+3)2} \le \sum_{i\in\{2,3\}}(\bar{v}_i - v_i)$$
$$v_3 \le p_{31} \le \bar{v}_3 - v_3$$
$$v_1 - v_1 \le p_{12} \le v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\bar{v}_1 - v_1 = p_{12}$$

Given the above price, $p_{31} + p_{(1+2)1} + p_{(2+3)2}$ cannot exceed $\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_i - v_i)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, the constraints $\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2} (\bar{v}_i - v_i)$, $\sum_{i \in \{2,3\}} v_i \le p_{(2+3)2} \le \sum_{i \in \{2,3\}} (\bar{v}_i - v_i), v_3 \le p_{31} \le \bar{v}_3 - v_3$ are more stringent than the non-negative surplus constraint on $p_{31} + p_{(1+2)1} + p_{(2+3)2}$. Therefore, $p_{(1+2)1} = \sum_{i=1}^{2} (\bar{v}_i - v_i), p_{(2+3)2} = \sum_{i \in \{2,3\}} (\bar{v}_i - v_i)$, $p_{31} = \bar{v}_3 - v_3$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, we have $p_{31} + p_{12} + p_{(2+3)2} = \sum_{i=1}^{3} \bar{v}_i - \sum_{i=1}^{2} (\bar{v}_i - v_i) - (\bar{v}_1 - v_1)$. Therefore, in the Nash Bargaining Solution, p_{31} maximizes $(p_{31} - v_3)\left(\sum_{i=1}^{3} \bar{v}_i - \sum_{i=1}^{2} (\bar{v}_i - v_i) - (\bar{v}_1 - v_1) - p_{31} - \sum_{i=1}^{3} v_i \right)$ and $p_{12} + p_{(2+3)2}$ maximizes $\left(p_{12} + p_{(2+3)2} - \sum_{i=1}^{3} v_i \right)\left(\sum_{i=1}^{3} \bar{v}_i - \sum_{i=1}^{2} (\bar{v}_i - v_i) - (\bar{v}_1 - v_1) - p_{12} - p_{(2+3)2} - v_3 \right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{32} + p_{(1+2)1} = \frac{\bar{v}_3 + 2v_1 + v_2}{2}$ and $p_{(2+3)2} = \frac{\bar{v}_3 + \bar{v}_2}{2}$.

13. Stage 1 strategy is (D, D)

This case is similar to (C, C). Since product 2 is sold separately by each seller, product 2 from the two sellers form a competing pair. Since they are complements, using Proposition 1(a), we have $p_{21} = p_{22} = \frac{\bar{v}_2}{2}$.

For the bundles with products 1 and 3, the case reduces to the two-product bundle case. The bundles can be substitutes or complements. So, using Proposition 1(b), we have the following result:

$$p_{(1+3)2} = p_{(1+3)1} = \sum_{i \in \{1,3\}} (\bar{v}_i - v_i) \; if \; 2 \sum_{i \in \{1,3\}} v_i \le \sum_{i \in \{1,3\}} \bar{v}_i$$

$$p_{(1+3)2} = p_{(1+3)1} = \frac{\sum_{i \in \{1,3\}} \bar{v}_i}{2} \; if \; 2 \sum_{i \in \{1,3\}} v_i > \sum_{i \in \{1,3\}} \bar{v}_i$$

14. Stage 1 strategy is (D, E)

Again, this case is similar to (C,E) except the products bundled by 1 are different in the two cases. Applying the same logic as that for (C,E) we get the following equilibrium prices.

(a) $\bar{v}_1 + \bar{v}_3 \le 2(v_1 + v_3)$

$\sum_{i \in \{1,3\}} (\bar{v}_i - v_i) = p_{(1+3)1}$

$\bar{v}_1 - v_1 = p_{12}$

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, the constraints $\sum_{i \in \{2,3\}} v_i \le p_{(2+3)2} \le \sum_{i \in \{2,3\}} (\bar{v}_i - v_i), v_3 \le p_{21} \le \bar{v}_2 - v_2$ are more stringent than the non-negative surplus constraint on $p_{21} + p_{12} + p_{(2+3)2}$. Therefore, $p_{(2+3)2} = \sum_{i \in \{2,3\}} (\bar{v}_i - v_i), p_{31} = \bar{v}_3 - v_3$.

If $\bar{v}_1+\bar{v}_2+\bar{v}_3>2(v_1+v_2+v_3)$, we have $p_{21}+p_{12}+p_{(2+3)2}=\sum_{i=1}^{3}\bar{v}_i-\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-(\bar{v}_1-v_1)$. Therefore, in the Nash Bargaining Solution, p_{21} maximizes $\left(p_{21}+p_{(2+3)2}=\sum_{i=1}^{3}v_i\right)\left(\sum_{i=1}^{3}\bar{v}_i-\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-(\bar{v}_1-v_1)-p_{12}-p_{(2+3)2}-v_2\right)$ and $p_{21}+p_{(2+3)2}$ maximizes $\left(p_{12}+p_{(2+3)2}=\sum_{i=1}^{3}v_i\right)\left(\sum_{i=1}^{3}\bar{v}_i-\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-(\bar{v}_1-v_1)-p_{12}-p_{(2+3)2}-v_2\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{21}=\frac{\bar{v}_2-\bar{v}_1+v_1}{2}$ and $p_{12}+p_{(2+3)2}=\frac{\bar{v}_2+3v_1+2v_3-\bar{v}_1}{2}$.

(b) $\bar{v}_1+\bar{v}_3>2(v_1+v_3)$

$\bar{v}_1-v_1=p_{12}$

Given the above price, $p_{21}+p_{(1+3)1}+p_{(2+3)2}$ cannot exceed $\sum_{i=1}^{3}\bar{v}_i-(\bar{v}_1-v_1)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1+\bar{v}_2+\bar{v}_3\le 2(v_1+v_2+v_3)$, , the constraints $\sum_{i\in\{1,3\}}v_i\le p_{(1+3)1}\le\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)$, $\sum_{i\in\{2,3\}}v_i\le p_{(2+3)2}\le\sum_{i\in\{2,3\}}(\bar{v}_i-v_i)$, $v_2\le p_{21}\le\bar{v}_2-v_3$ are more stringent than the non-negative surplus constraint on $p_{21}+p_{(1+3)1}+p_{(2+3)2}$. Therefore, $p_{(1+3)1}=\sum_{i\in\{1,3\}}(\bar{v}_i-v_i), p_{(2+3)2}=\sum_{i\in\{2,3\}}(\bar{v}_i-v_i), p_{21}=\bar{v}_2-v_2$.

If $\bar{v}_1+\bar{v}_2+\bar{v}_3>2(v_1+v_2+v_3)$, we have $p_{21}+p_{(1+3)1}+p_{(2+3)2}=\sum_{i=1}^{3}\bar{v}_i-(\bar{v}_1-v_1)$. Therefore, in the Nash Bargaining Solution, $p_{21}+p_{(1+3)1}$ maximizes $\left(p_{21}+p_{(1+3)1}-\sum_{i=1}^{3}v_i\right)\left(\sum_{i=1}^{3}\bar{v}_i-(\bar{v}_1-v_1)-p_{21}-p_{(1+3)1}-\sum_{i=2}^{3}v_i\right)$ and $p_{(2+3)2}$ maximizes $\left(p_{(2+3)2}-\sum_{i=2}^{3}v_i\right)\left(\sum_{i=1}^{3}\bar{v}_i-(\bar{v}_1-v_1)-p_{(2+3)2}-\sum_{i=1}^{3}v_i\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{21}+p_{(1+3)1}=\frac{\bar{v}_3+2v_1+\bar{v}_2}{2}$ and $p_{(2+3)2}=\frac{\bar{v}_1+\bar{v}_2}{2}$.

15. Stage 1 strategy is (E, E)

This case is similar to (C, C) or (D, D). Applying the same logic as that for (C, C), we get $p_{11}=p_{12}=\bar{v}_1-v_1, p_{(2+3)2}=p_{(2+3)1}=\frac{\bar{v}_2+\bar{v}_3}{2}$.

The price equilibrium for other 10 possible strategy pairs can be determined from the above using symmetry.

Bundling Equilibrium in the First Stage

The analysis of the second stage shows that the price equilibrium differs depending on the relative product valuation parameters. Specifically, the price equilibrium depends on the combination of the following conditions:

(i) $\bar{v}_1+\bar{v}_2+\bar{v}_3>2(v_1+v_2+v_3)$ or $\bar{v}_1+\bar{v}_2+\bar{v}_3\le(v_1+v_2+v_3)$

(ii) $\bar{v}_1+\bar{v}_2>2(v_1+v_2)$ or $\bar{v}_1+\bar{v}_2\le(v_1+v_2)$

(iii) $\bar{v}_1+\bar{v}_3>2(v_1+v_3)$ or $\bar{v}_1+\bar{v}_3\le(v_1+v_3)$

Since the payoff matrix for stage 1 analysis depends on the combination, and there are numerous combinations, we show the stage 1 analysis for one of the combinations. The analysis for other combinations is similar to the one presented, and the results for those combinations are qualitatively similar to the one presented here.

Assume $\bar{v}_1+\bar{v}_2+\bar{v}_3\le(v_1+v_2+v_3), \bar{v}_1+\bar{v}_2\le(v_1+v_2)$, and $\bar{v}_1+\bar{v}_3\le(v_1+v_3)$

We have the following payoff matrix for the product policy game in the first stage for the above combination. In each cell, the first expression is the payoff for seller 1 and the second expression is the payoff for seller 2.

S1 / S2	**A**	**B**	**C**	**D**	**E**
A	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2},\\ (\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{\bar{v}_3}{2}+\sum_{i=1}^{2}(\bar{v}_i-v_i)-\varepsilon,\\ \frac{\bar{v}_3}{2}+\sum_{i=1}^{2}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon,\\ \frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$
B	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2},\\ (\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}\end{pmatrix}$
C	$\begin{pmatrix}\frac{\bar{v}_3}{2}+\sum_{i=1}^{2}(\bar{v}_i-v_i)-\varepsilon,\\ \frac{\bar{v}_3}{2}+\sum_{i=1}^{2}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2},\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{2}(\bar{v}_i-v_i)+\frac{v_3}{2}-\varepsilon,\\ \sum_{i=1}^{2}(\bar{v}_i-v_i)+\frac{v_3}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$
D	$\begin{pmatrix}\frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon,\\ \frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2},\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon,\\ \frac{\bar{v}_2}{2}+\sum_{i\in\{1,3\}}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$
E	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2},\\ (\bar{v}_1-v_1)+\sum_{i\in\{2,3\}}\frac{\bar{v}_i}{2}-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}\sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon,\\ \sum_{i=1}^{3}(\bar{v}_i-v_i)-\varepsilon\end{pmatrix}$	$\begin{pmatrix}(\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}-\varepsilon,\\ (\bar{v}_1-v_1)+\sum_{i=2}^{3}\frac{\bar{v}_i}{2}\end{pmatrix}$

From the above payoff matrix, it is easy to see that (A, A) is a Pareto-optimal Nash equilibrium because $\bar{v}_1 \le 2v_1$, $\bar{v}_2 > 2v_2$, and $\bar{v}_3 > 2v_3$. We also have (C, D), (C, E), (D, C), (D, E), (E, C), and (E, D) as the other Nash equilibria that have the same equilibrium payoff as (A, A). We note that all these equilibria involve both sellers bundling at least two products, and at least one seller bundling the substitutor product 1 with one of the other two complementor products.

(d) One Pair Is Complementor and the Other Two Are Substitutors

Without loss of generality, we assume that competing pair 1 is the complementor and the other two pairs are substitutors. That is, $\bar{v}_1 > 2v_1, \bar{v}_2 \le 2v_2$, and $\bar{v}_3 \le 2v_3$.

Price Equilibrium in Second Stage

1. Stage 1 strategy is (A, A)

Same as (A, A) for scenario (c) discussed previously.

2. Stage 1 strategy is (A, B)

For seller 2, the feasible and non-dominated region for p_{12} is given by $\left(v_1, \bar{v}_1 - v_1\right)$, the feasible and non-dominated region for p_{22} is given by $\left(\bar{v}_2 - v_2, v_2\right)$, and the feasible and non-dominated region for p_{32} is given by $\left(\bar{v}_3 - v_3, v_3\right)$.

For seller 1, we need to consider two cases: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$.

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$

The following should hold in the equilibrium:

$$
\begin{aligned}
&\sum_{i=1}^{3}\left(\bar{v}_i - v_i\right) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i \\
&v_1 \le p_{12} \le \bar{v}_1 - v_1 \\
&\bar{v}_2 - v_2 \le p_{22} \le v_2 \\
&\bar{v}_3 - \bar{v}_3 \le p_{32} \le v_3
\end{aligned}
$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$
\begin{aligned}
&\bar{v}_2 - v_2 = p_{22} \\
&\bar{v}_3 - v_3 = p_{32} \\
&v_1 \le p_{(1+2+3)1} - \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right) \le \left(\bar{v}_1 - v_1\right)
\end{aligned}
$$

Using the Nash Bargaining Solution model discussed, we obtain $p_{(1+2+3)1} - \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right) = p_{12} = \frac{\bar{v}_1}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$

The following should hold in the equilibrium:

$$
\begin{aligned}
&\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\bar{v}_i - v_i\right) \\
&v_1 \le p_{12} \le \bar{v}_1 - v_1 \\
&\bar{v}_2 - v_2 \le p_{22} \le v_2 \\
&\bar{v}_3 - \bar{v}_3 \le p_{32} \le v_3
\end{aligned}
$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\bar{v}_2 - v_2 = p_{22}$$
$$\bar{v}_3 - v_3 = p_{32}$$
$$v_1 \le p_{(1+2+3)1} - \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right) \le \left(\bar{v}_1 - v_1\right)$$

Using the Nash Bargaining Solution model discussed, we obtain $p_{(1+2+3)1} - \sum_{i=2}^{3}\left(\bar{v}_i - v_i\right) = p_{12} = \frac{\bar{v}_1}{2}$.

3. Stage 1 strategy is (A, C)

For seller 1, we need to consider two cases: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$.

For seller 2, the feasible and non-dominated regions for p_{32} is given by $\left(\bar{v}_3 - v_3, v_3\right)$, and the feasible and non-dominated region for $p_{(1+2)2}$ depends on whether $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$ or $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$. So, the price equilibrium depends on which of the following conditions are satisfied: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$, (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$, (iii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$. Note that the conditions $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ are not possible because $\bar{v}_3 \le 2v_3$ in this scenario.

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3}\left(\bar{v}_1 - v_i\right) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$
$$\sum_{i=1}^{2}\left(\bar{v}_1 - v_i\right) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$
$$\bar{v}_3 - v_3 \le p_{32} \le v_3$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) = p_{(1+2)2}$$
$$\bar{v}_3 - v_3 = p_{32}$$
$$p_{(1+2+3)1} = \sum_{i=1}^{3}\left(\bar{v}_i - v_i\right)$$

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\bar{v}_1 - v_i\right)$$
$$\sum_{i=1}^{2}\left(\bar{v}_1 - v_i\right) \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$
$$\bar{v}_3 \le p_{32} \le \bar{v}_3 - v_3$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) = p_{(1+2)2}$$

$$v_3 \le p_{(1+2+3)1} - \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) \le \bar{v}_3 - v_3$$

Using the Nash Bargaining Solution model, we obtain $p_{(1+2+3)1} - \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) = p_{32} = \frac{\bar{v}_3}{2}$.

(iii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \ge 2\left(v_1 + v_2\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3}\left(\bar{v}_1 - v_i\right)$$

$$\sum_{i=1}^{2} v_i \le p_{(1+2)2} \le \sum_{i=1}^{2}\left(\bar{v}_1 - v_i\right)$$

$$\bar{v}_3 \le p_{32} \le \bar{v}_3 - v_3$$

Using the Nash Bargaining Solution model, we obtain $p_{(1+2+3)1} = p_{(1+2)2} + p_{32} = \sum_{i=1}^{2}\frac{\bar{v}_i}{2}$.

4. Stage 1 strategy is (A, D)

The proof is similar to the (A, C) case; the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (A, C) case. Applying the same logic as in the (A, C) case, we can show that there are three possible combinations of conditions to consider, and that the equilibrium prices will be the following:

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$ and $\bar{v}_1 + \bar{v}_2 \le 2\left(v_1 + v_2\right)$

$$\sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right) = p_{(1+3)2}$$

$$\bar{v}_2 - v_2 = p_{22}$$

$$p_{(1+2+3)1} = \sum_{i=1}^{3}\left(\bar{v}_i - v_i\right)$$

(ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$ and $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

$$\sum_{i\in\{1,3\}} (\bar{v}_i - v_i) = p_{(1+3)2}$$

$$p_{(1+2+3)1} - \sum_{i=1}^{3} (\bar{v}_i - v_i) = p_{22} = \frac{\bar{v}_2}{2}$$

(iii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$ and $\bar{v}_1 + \bar{v}_2 \ge 2(v_1 + v_2)$

$$p_{(1+2+3)1} = p_{(1+2)2} + p_{32} = \sum_{i=1}^{3} \frac{\bar{v}_i}{2}$$

5. Stage 1 strategy is (A, E)

For seller 1, we need to consider two cases: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, and (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$.
For seller 2, the feasible and non-dominated regions for p_{12} is given by $(v_1, \bar{v}_1 - v_1)$, and the feasible and non-dominated region for $p_{(2+3)2}$ is given by $\left(\sum_{i=1}^{3} (\bar{v}_i - v_i), \sum_{i=2}^{3} v_i\right)$.

So, the price equilibrium depends on which of the following conditions are satisfied: (i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, and (ii) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$.

(a) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} (\bar{v}_i - v_i) \le p_{(1+2+3)1} \le \sum_{i=1}^{3} v_i$$

$$\sum_{i=2}^{3} (\bar{v}_i - v_i) \le p_{(2+3)2} \le \sum_{i=2}^{3} v_i$$

$$v_1 \le p_{12} \le \bar{v}_1 - v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$p_{(1+2+3)1} = \sum_{i=2}^{3} (\bar{v}_i - v_i)$$

$$\sum_{i=2}^{3} (\bar{v}_i - v_i) = p_{(2+3)2}$$

$$p_{12} = \bar{v}_1 - v_1$$

(b) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$.

The following should hold in the equilibrium:

$$\sum_{i=1}^{3} v_i \le p_{(1+2+3)1} \le \sum_{i=1}^{3} (\bar{v}_i - v_i)$$
$$\sum_{i=2}^{3} (\bar{v}_i - v_i) \le p_{(2+3)2} \le \sum_{i=2}^{3} v_i$$
$$v_1 \le p_{12} \le \bar{v}_1 - v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=2}^{3} (\bar{v}_i - v_i) = p_{(2+3)2}$$

Using the Nash Bargaining Solution model , we obtain $p_{(1+2+3)1} - \sum_{i=2}^{3} (\bar{v}_i - v_i) = p_{12} = \frac{\bar{v}_1}{2}$.

6. Stage 1 strategy is (B, B)

Using Proposition 3 (a) , we have each seller's price for product 1 as $\bar{v}_1 - v_1$, for product 2 as $\frac{\bar{v}_2}{2}$, and for product 3 as $\frac{\bar{v}_3}{2}$.

7. Stage 1 strategy is (B, C)

For seller 1, the feasible and non-dominated region for p_{11} is given by $(v_1, \bar{v}_1 - v_1)$, that for p_{21} is given by $(\bar{v}_2 - v_2, v_2)$, and that for p_{31} is given by $(\bar{v}_3 - v_3, v_3)$. For seller 2, the feasible and non-dominated regions for p_{32} is given by $(\bar{v}_3 - v_3, v_3)$. The feasible and non-dominated regions for $p_{(1+2)2}$ depends on whether $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$ or $\bar{v}_1 + \bar{v}_2 \ge 2(v_1 + v_2)$.

So, the price equilibrium depends on which of the following conditions are satisfied: (i) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$, (ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$.

(i) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$v_1 \le p_{11} \le \bar{v}_1 - v_1$$
$$\bar{v}_2 - v_2 \le p_{21} \le v_2$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$\sum_{i=1}^{2} \bar{v}_i - v_i \le p_{(1+2)2} \le \sum_{i=1}^{2} v_i$$
$$\bar{v}_3 - v_3 \le p_{32} \le v_3$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\bar{v}_2 - v_2 = p_{21}$$
$$\bar{v}_3 - v_3 = p_{31}$$
$$\bar{v}_3 - v_3 = p_{32}$$
$$v_1 \le p_{(1+2)2} - \bar{v}_2 + v_2 \le \bar{v}_1 - v_1$$

Using the Nash Bargaining Solution model, we obtain $p_{(1+2)2} - \bar{v}_2 + v_2 = p_{11} = \frac{\bar{v}_1}{2}$.

(ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$

The following should hold in the equilibrium:

$$\begin{aligned}
&v_1 \le p_{11} \le \bar{v}_1 - v_1\\
&\bar{v}_2 - v_2 \le p_{21} \le v_2\\
&\bar{v}_3 - v_3 \le p_{31} \le v_3\\
&\sum_{i=1}^{2} v_i \le p_{(1+2)2} \le \sum_{i=1}^{2} \bar{v}_i - v_i\\
&\bar{v}_3 - v_3 \le p_{32} \le v_3
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\begin{aligned}
&\bar{v}_2 - v_2 = p_{21}\\
&\bar{v}_3 - v_3 = p_{31}\\
&\bar{v}_3 - v_3 = p_{32}\\
&v_1 \le p_{(1+2)2} - \bar{v}_2 + v_2 \le \bar{v}_1 - v_1
\end{aligned}$$

Using the Nash Bargaining Solution model, we obtain $p_{(1+2)2} - \bar{v}_2 + v_2 = p_{11} = \frac{\bar{v}_1}{2}$.

8. Stage 1 strategy is (B, D)

The proof is similar to the (B, C) case ; the only difference is that seller 2 bundles product 3 with product 1 in this case, whereas it was product 2 that was bundled with product 1 in (B, C) case. Applying the same logic as in the (B, C) case, we can show the following result: $\bar{v}_2 - v_2 = p_{21}, \bar{v}_3 - v_3 = p_{31}, \bar{v}_2 - v_2 = p_{32}$ and $p_{(1+3)2} - \bar{v}_3 + v_3 = p_{11} = \frac{\bar{v}_1}{2}$.

9. Stage 1 strategy is (B, E)

For seller 1, the feasible and non-dominated region for p_{11} is given by $(v_1, \bar{v}_1 - v_1)$, that for p_{21} is given by $(\bar{v}_2 - v_2, v_2)$, and that for p_{12} is given by $(\bar{v}_3 - v_3, v_3)$. For seller 2, the feasible and non-dominated regions for p_{12} is given by $(v_1, \bar{v}_1 - v_1)$. The feasible and non-dominated regions for $p_{(2+3)2}$ is given by $\left(\sum_{i=2}^{3} \bar{v}_i - v_i, \sum_{i=2}^{3} v_i\right)$.

The following should hold in the equilibrium:

$$\begin{aligned}
&v_1 \le p_{11} \le \bar{v}_1 - v_1\\
&\bar{v}_2 - v_2 \le p_{21} \le v_2\\
&\bar{v}_3 - v_3 \le p_{31} \le v_3\\
&\sum_{i=1}^{2} \bar{v}_i - v_i \le p_{(2+3)2} \le \sum_{i=1}^{2} v_i\\
&v_1 \le p_{12} \le \bar{v}_1 - v_1
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\bar{v}_2 - v_2 = p_{21}$$
$$\bar{v}_3 - v_3 = p_{31}$$
$$\sum_{i=2}^{3} \bar{v}_i - v_i = p_{(2+3)2}$$

Using the Nash Bargaining Solution model we obtain $p_{11} = p_{21} = \frac{\bar{v}_1}{2}$.

10. Stage 1 strategy is (C, C)

Since product 3 is sold separately by each seller, product 3 from the two sellers form a competing pair. Since they are complements, using Proposition 1(a), we have $p_{31} = p_{32} = \frac{\bar{v}_3}{2}$.

For the bundles with products 1 and 2, the case reduces to the two-product bundle case. The bundles can be substitutes or complements. So, using Proposition 1(b), we have the following result:

$$p_{(1+2)2} = p_{(1+2)1} = \sum_{i=1}^{2} (\bar{v}_i - v_i) \; if \; 2\sum_{i=1}^{2} v_i \le \sum_{i=1}^{2} \bar{v}_i$$

11. Stage 1 strategy is (C, D)

For seller 1, the feasible and non-dominated region for p_{31} is given by $(v_3, \bar{v}_3 - v_3)$. The feasible and non-dominated region for $p_{(1+2)1}$ depends on whether $2\sum_{i=1}^{2} v_i \le \sum_{i=1}^{2} \bar{v}_i$ or $2\sum_{i=1}^{2} v_i > \sum_{i=1}^{2} \bar{v}_i$. For seller 2, the feasible and non-dominated region for p_{22} is given by $(\bar{v}_2 - v_2, v_2)$. The feasible and non-dominated region for $p_{(1+3)2}$ depends on if $2\sum_{i\in\{1,3\}} v_i \le \sum_{i\in\{1,3\}} \bar{v}_i$ or $2\sum_{i\in\{1,3\}} v_i > \sum_{i\in\{1,3\}} \bar{v}_i$.

So, the price equilibrium depends on which of the following conditions are satisfied: (i) $v_1 + \bar{v}_2 \le 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 \le 1(v_1 + v_3)$, (ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 \le 2(v_1 + v_3)$, (iii) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 > 2(v_1 + v_3)$, (iv) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 > 2(v_1 + v_3)$

(i) $v_1 + \bar{v}_2 \le 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 \le 1(v_1 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2} (\bar{v}_i - v_i) \le p_{(1+2)1} \le \sum_{i-1}^{2} v_i$$
$$\sum_{i\in\{1,3\}} (\bar{v}_i - v_i) \le p_{(1+3)2} \le \sum_{i\in\{1,3\}} v_i$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$\bar{v}_2 - v_2 \le p_{22} \le v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}(\bar{v}_i - v_i) = p_{(1+2)1}$$
$$\sum_{i\in\{1,3\}}(\bar{v}_i - v_i) \le p_{(1+3)2}$$
$$\bar{v}_3 - v_3 = p_{31}$$
$$\bar{v}_2 - v_2 = p_{22}$$

(ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 \le 2(v_1 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i-1}^{2}(\bar{v}_i - v_i)$$
$$\sum_{i\in\{1,3\}}(\bar{v}_i - v_i) \le p_{(1+3)2} \le \sum_{i\in\{1,3\}} v_i$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$\bar{v}_2 - v_2 \le p_{22} \le v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i\in\{1,3\}}(\bar{v}_i - v_i) \le p_{(1+3)2}$$
$$\bar{v}_3 - v_3 = p_{31}$$
$$\bar{v}_2 - v_2 = p_{22}$$

Given the above prices, $p_{(1+2)1}$ cannot exceed $\sum_{i=1}^{3}\bar{v}_i - \sum_{i\in\{1,3\}}(\bar{v}_i - v_i) - (\bar{v}_3 - v_3) - (\bar{v}_2 - v_2)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, the constraint $\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2}(\bar{v}_i - v_i)$ is more stringent than the non-negative surplus constraint on $p_{(1+2)1}$. Therefore, $p_{(1+2)1} = \sum_{i=1}^{2}(\bar{v}_i - v_i)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, then $p_{(1+2)1} = \sum_{i=1}^{3}\bar{v}_i - \sum_{i\in\{1,3\}}(\bar{v}_i - v_i) - (\bar{v}_3 - v_3) - (\bar{v}_2 - v_2)$.

(iii) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$ and $\bar{v}_1 + \bar{v}_3 > 2(v_1 + v_3)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) \le p_{(1+2)1} \le \sum_{i-1}^{2} v_i$$
$$\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right)$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$\bar{v}_2 - v_2 \le p_{22} \le v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller:

$$\sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) = p_{(1+2)1}$$
$$\bar{v}_3 - v_3 = p_{31}$$
$$\bar{v}_2 - v_2 = p_{22}$$

Given the above prices, $p_{(1+3)2}$ cannot exceed $\sum_{i=1}^{3}\bar{v}_i - \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) - \left(\bar{v}_3 - v_3\right) - \left(\bar{v}_2 - v_2\right)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2\left(v_1 + v_2 + v_3\right)$, the constraint $\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right)$ is more stringent than the non-negative surplus constraint on $p_{(1+3)2}$. Therefore, $p_{(1+3)2} = \sum_{i\in[1,3]}\left(\bar{v}_i - v_i\right)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$, then $p_{(1+3)2} = \sum_{i=1}^{3}\bar{v}_i - \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right) - \left(\bar{v}_3 - v_3\right) - \left(\bar{v}_2 - v_2\right)$.

(iv) $\bar{v}_1 + \bar{v}_2 > 2\left(v_1 + v_2\right)$ and $\bar{v}_1 + \bar{v}_3 > 2\left(v_1 + v_3\right)$

The following should hold in the equilibrium:

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i-1}^{2}\left(\bar{v}_i - v_i\right)$$
$$\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right)$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$\bar{v}_2 - v_2 \le p_{22} \le v_2$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices:

$$\bar{v}_3 - v_3 = p_{31}$$
$$\bar{v}_2 - v_2 = p_{22}$$

Given the above prices, $p_{(1+3)2} + p_{(1+2)1}$ cannot exceed $\sum_{i=1}^{3}\bar{v}_i - \left(\bar{v}_3 - v_3\right) - \left(\bar{v}_2 - v_2\right)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2\left(v_1 + v_2 + v_3\right)$, the constraints $\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right)$, $\sum_{i\in\{1,3\}} v_i \le p_{(1+3)2} \le \sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right)$ are more stringent than the non-negative surplus constraint on $p_{(1+3)2} + p_{(1+2)1}$. Therefore, $p_{(1+2)1} = \sum_{i=1}^{2}\left(\bar{v}_i - v_i\right)$, $p_{(1+3)2} = \sum_{i\in\{1,3\}}\left(\bar{v}_i - v_i\right)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, then $p_{(1+3)2} + p_{(1+2)1} = \sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - (\bar{v}_2 - v_2)$.

Therefore, in the Nash Bargaining Solution, $p_{(1+3)2}$ maximizes $\left(p_{(1+3)2} - \sum_{i\in\{1,3\}} v_i\right)\left(\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - (\bar{v}_2 - v_2) - p_{(1+3)2} - \sum_{i=1}^{2} v_i\right)$ and $p_{(1+2)1}$ maximizes $\left(p_{(1+2)1} - \sum_{i=1}^{2} v_i\right)\left(\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - (\bar{v}_2 - v_2) - p_{(1+2)1} - \sum_{i\in\{1,3\}} v_i\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{(1+3)2} = v_3 + \frac{\bar{v}_1}{2}$ and $p_{(1+2)1} = v_2 + \frac{\bar{v}_1}{2}$.

12. Stage 1 strategy is (C, E)

For seller 1, the feasible and non-dominated region for p_{31} is given by $(\bar{v}_3 - v_3, v_3)$. The feasible and non-dominated region for $p_{(1+2)1}$ depends on whether $2\sum_{i=1}^{2} v_i \le \sum_{i=1}^{2} \bar{v}_i$ or $2\sum_{i=1}^{2} v_i > \sum_{i=1}^{2} \bar{v}_i$. For seller 2, the feasible and non-dominated region for p_{12} is given by $(v_i, \bar{v}_i - v_i)$. The feasible and non-dominated region for $p_{(2+3)2}$ is given by $\left(\sum_{i\in\{2,3\}} (\bar{v}_i - v_i), \sum_{i\in\{2,3\}} v_i\right)$.

So, the price equilibrium depends on which of the following conditions are satisfied: (i) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$, (ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$

(i) $\bar{v}_1 + \bar{v}_2 \le 2(v_1 + v_2)$

The following should hold in the equilibrium.

$$\begin{aligned}
&\sum_{i=1}^{2} (\bar{v}_i - v_i) \le p_{(1+2)1} \le \sum_{i-1}^{2} v_i \\
&\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) \le p_{(2+3)2} \le \sum_{i\in\{1,3\}} v_i \\
&\bar{v}_3 - v_3 \le p_{31} \le v_3 \\
&v_1 \le p_{12} \le \bar{v}_1 - v_1
\end{aligned}$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\begin{aligned}
&\sum_{i=1}^{2} (\bar{v}_i - v_i) = p_{(1+2)1} \\
&\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) = p_{(2+3)2} \\
&\bar{v}_3 - v_3 = p_{31}
\end{aligned}$$

Given the above prices, p_{12} cannot exceed $\sum_{i=1}^{3} \bar{v}_i - \sum_{i=1}^{2} (\bar{v}_i - v_i) - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3)$, the constraint $v_1 \le p_{12} \le \bar{v}_1 - v_1$ is more stringent than the non-negative surplus constraint on p_{11}. Therefore, $p_{12} = \bar{v}_1 - v_1$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, then $p_{12} = \sum_{i=1}^{3} \bar{v}_i - \sum_{i=1}^{2} (\bar{v}_i - v_i) - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$.

(ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$

The following should hold in the equilibrium.

$$\sum_{i=1}^{2} v_i \le p_{(1+2)1} \le \sum_{i-1}^{2} (\bar{v}_i - v_i)$$
$$\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) \le p_{(2+3)2} \le \sum_{i\in\{2,3\}} v_i$$
$$\bar{v}_3 - v_3 \le p_{31} \le v_3$$
$$v_1 \le p_{12} \le \bar{v}_1 - v_1$$

Since in the equilibrium, each seller will set prices so that every offering is sold, the following characterizes the equilibrium prices for each offering from each seller.

$$\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) = p_{(2+3)2}$$
$$\bar{v}_3 - v_3 = p_{31}$$

Given the above prices, $p_{12} + p_{(1+2)1}$ cannot exceed $\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, the constraints $\sum_{i=1}^{2} \bar{v}_i \le p_{(1+2)1} \le \sum_{i=1}^{2} (\bar{v}_i - v_i), v_1 \le p_{12} \le \bar{v}_1 - v_1$ are more stringent than the non-negative surplus constraint on $p_{12} + p_{(1+2)1}$. Therefore, $p_{(1+2)1} = \sum_{i=1}^{2} (\bar{v}_i - v_i), p_{12} = (\bar{v}_1 - v_1)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, then $p_{12} + p_{(1+2)1} = \sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$.

Therefore, in the Nash Bargaining Solution, $p_{(1+2)1}$ maximizes $\left(p_{(1+2)1} - \sum_{i\in\{1,2\}} v_i\right)\left(\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i) - p_{(1+2)1} - v_1\right)$ and p_{12} maximizes $(p_{12} - v_1)\left(\sum_{i=1}^{3} \bar{v}_i - (\bar{v}_3 - v_3) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i) - p_{12} - \sum_{i\in\{1,2\}} v_i\right)$. Solving the above optimization problems simultaneously gives the following solution: $p_{(1+2)1} = \frac{\bar{v}_1 + 2v_3 + 2v_2 - \bar{v}_3}{2}$ and $p_{12} = \frac{\bar{v}_1 + 2v_3 - \bar{v}_3}{2}$.

13. Stage 1 strategy is (D, D)

This case is similar to (C, C). Since product 2 is sold separately by each seller, product 2 from the two sellers form a competing pair. Since they are substitutes, using Proposition 1(a), we have $p_{21} = p_{22} = \bar{v}_2 - v_1$.

For the bundles with products 1 and 3, the case reduces to the two-product bundle case. The bundles can be substitutes or complements. So, using Proposition 1(b), we have the following result:

$$p_{(1+3)2} = p_{(1+3)1} = \sum_{i\in\{1,3\}} (\bar{v}_i - v_i)\, if\ 2\sum_{i\in\{1,3\}} v_i \le \sum_{i\in\{1,3\}} \bar{v}_i$$
$$p_{(1+3)2} = p_{(1+3)1} = \frac{\sum_{i\in\{1,3\}} \bar{v}_i}{2}\, if\ 2\sum_{i\in\{1,3\}} v_i > \sum_{i\in\{1,3\}} \bar{v}_i$$

14. Stage 1 strategy is (D, E)

Again, this case is similar to (C, E) except the products bundled by 1 are different in the two cases. Applying the same logic as that for (C, E) we get the following equilibrium prices:

(1) $\bar{v}_1 + \bar{v}_3 \le 2(v_1 + v_2)$

$$\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) = p_{(1+2)1}$$
$$\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) = p_{(2+3)2}$$
$$\bar{v}_2 - v_2 = p_{21}$$

Given the above prices, p_{12} cannot exceed $\sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} (\bar{v}_i - v_i) - (\bar{v}_2 - v_2) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3), p_{12} = \bar{v}_1 - v_1$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, then $p_{12} = \sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} (\bar{v}_i - v_i) - (\bar{v}_2 - v_2) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$.

(ii) $\bar{v}_1 + \bar{v}_3 > 2(v_1 + v_3)$

$$\sum_{i\in\{2,3\}} (\bar{v}_i - v_i) = p_{(2+3)2}$$
$$\bar{v}_2 - v_2 = p_{21}$$

Given the above prices, $p_{12} + p_{(1+3)1}$ cannot exceed $\sum_{i=1}^{3} \bar{v}_i - \sum_{i\in\{1,3\}} \bar{v}_i - (\bar{v}_2 - v_2) - \sum_{i\in\{2,3\}} (\bar{v}_i - v_i)$ in order to ensure non-negative surplus for the consumer by buying these products.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le 2(v_1 + v_2 + v_3), p_{(1+3)1} = \sum_{i\in\{1,3\}} (\bar{v}_i - v_i), p_{12} = (\bar{v}_i - v_i)$.

If $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$, then $p_{(1+3)1} = \dfrac{\bar{v}_1 + 2\bar{v}_3 + 2v_2 - \bar{v}_2}{2}$ and $p_{12} = \dfrac{\bar{v}_1 + v_2 - \bar{v}_2}{2}$.

15. Stage 1 strategy is (E, E)

This case is similar to (C, C) or (D, D). Applying the same logic as that for (C, C), we get $p_{11} = p_{12} = \dfrac{\bar{v}_1}{2}$, $p_{(2+3)2} = p_{(2+3)1} = \bar{v}_2 - v_2 + \bar{v}_3 - v_3$.

The price equilibrium for other 10 possible strategy pairs can be determined from the above using symmetry.

Bundling Equilibrium in the First Stage

The analysis of the second stage shows that the exact price equilibrium differs depending on the relative product valuation parameters. Specifically, the price equilibrium depends on the combination of the following conditions:

(i) $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 > 2(v_1 + v_2 + v_3)$ or $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le (v_1 + v_2 + v_3)$
(ii) $\bar{v}_1 + \bar{v}_2 > 2(v_1 + v_2)$ or $\bar{v}_1 + \bar{v}_2 \le (v_1 + v_2)$
(iii) $\bar{v}_1 + \bar{v}_3 > 2(v_1 + v_3)$ or $\bar{v}_1 + \bar{v}_3 \le (v_1 + v_3)$

Since the payoff matrix for stage 1 analysis depends on the combination, and there are numerous combinations, we show the stage 1 analysis for one of the combinations. The analysis for other combinations is similar to the one presented, and the results for those combinations are qualitatively similar to the one presented here.

Assume $\bar{v}_1 + \bar{v}_2 + \bar{v}_3 \le (v_1 + v_2 + v_3), \bar{v}_1 + \bar{v}_2 \le (v_1 + v_2)$, and $\bar{v}_1 + \bar{v}_3 \le (v_1 + v_3)$.

We have the following payoff matrix for the product policy game in the first stage for the above combination. In each cell, the first expression is the payoff for seller 1 and the second expression is the payoff for seller 2.

S1 / S2	**A**	**B**	**C**	**D**	**E**
A	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2}, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$
B	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2}, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$
C	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2}, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$
D	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2}, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$
E	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2}, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon, \\ \sum_{i=1}^{3}(\bar{v}_i - v_i) - \varepsilon \end{pmatrix}$	$\begin{pmatrix} \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon, \\ \sum_{i=2}^{3}(\bar{v}_1 - v_1) + \frac{\bar{v}_1}{2} - \varepsilon \end{pmatrix}$

From the above payoff matrix, it is easy to see that (A, A) is a Paerto-optimal Nash equilibrium because $\bar{v}_1 \le 2v_1, \bar{v}_2 > 2v_2$, and $\bar{v}_3 > 2v_3$. We also have (A, C), (A, D), (C, A), (C, D), (C, E), (D, A), (D, C), (D, D), (D, E), (E, C), and (E, D) as the other Nash equilibria that have the same equilibrium payoff as (A,A). We note that all these equilibria involve both sellers bundling at least two products, and at least one seller bundling the complementor product 1 with one of the other two complementor products.