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PLATFORM ECOSYSTEMS: HOW DEVELOPERS INVERT THE FIRM

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Appendix

Proof of Proposition 1

Recall from the model setup Eq. (5) that

$$\pi_p = V(1-\sigma) + \tfrac{1}{2}v(1-\delta)k(\sigma V)^{\alpha} + \delta\tfrac{1}{2}v(1-\delta)k^{1+\alpha}(\sigma V)^{\alpha^2} \tag{9}$$

The corresponding first-order conditions w.r.t. δ and σ become

$$0 = \tfrac{\partial \pi_p}{\partial \sigma} = -V + \tfrac{1}{2}v(1-\delta)\left[k\alpha\sigma^{\alpha-1}V^{\alpha} + \delta\alpha^2 k^{1+\alpha}(N_r)^{\alpha}\sigma^{\alpha^2-1}V^{\alpha^2}\right] \tag{10}$$

$$0 = \tfrac{\partial \pi_p}{\partial \delta} = -\tfrac{1}{2}vk(\sigma V)^{\alpha} + \tfrac{1}{2}v(1-\delta)k^{1+\alpha}(N_r)^{\alpha}(\sigma V)^{\alpha^2} - \delta\tfrac{1}{2}vk^{1+\alpha}(N_r)^{\alpha}(\sigma V)^{\alpha^2} \tag{11}$$

Table A1. Parameter Definitions

Var	Parameter Definition
σ	Share of platform (%) opened to developers
t, δ	Time until exclusionary period expires (discount $\delta = e^{-rt}$)
α	Technology in Cobb Douglas production
K	Coefficient of reuse
M_d, M_u	Market spillovers from developers and users, index sizes of network effects
N_d, N_u	Number of developers and users respectively
p	Price of individual developer applications $p = v(1 - \delta)$
ρ	Technological uncertainty; equal to $1 - \omega$
v	Value, per unit, of developer output
V	Standalone value of sponsor's platform
y_i	Output of a single developer in period *i* and input to developers in period *i* + 1 with $y_0 = \sigma V$ and $y_{i+1} = ky_{i+1}^{\alpha}$
ω	Probability of success for a given innovation; equal to $1 - \rho$

Multiply Eq. (10) by σ to get

$$0 = -\sigma V + \tfrac{1}{2} k\alpha v(1-\delta)\left[(\sigma V)^{\alpha} + \delta\alpha k^{\alpha}(N_r)^{\alpha}(\sigma V)^{\alpha^2}\right] \tag{12}$$

Denote

$$S := \sigma V \tag{13}$$

Then Eq. (12) becomes

$$S = \tfrac{1}{2} k\alpha v(1-\delta)\left[S^{\alpha} + \delta\alpha k^{\alpha}(N_r)^{\alpha} S^{\alpha^2}\right]$$

or

$$1 = \left(\alpha k S^{\alpha-1}\right)\tfrac{v}{2}(1-\delta)\left[1 + \delta\alpha\left(kN_r S^{\alpha-1}\right)^{\alpha}\right] \tag{14}$$

Similarly, Eq. (11) becomes

$$0 = \tfrac{1}{2} vkS^{\alpha} + \tfrac{1}{2} v(1-\delta)k^{1+\alpha}(N_r)^{\alpha} S^{\alpha^2} - \delta\tfrac{1}{2} vk^{1+\alpha}(N_r)^{\alpha} S^{\alpha^2}$$

Equivalently,

$$\delta = \tfrac{1}{2}\left[1 - \left(N_r kS^{\alpha-1}\right)^{-\alpha}\right] \tag{15}$$

Denote

$$M := kS^{\alpha-1} \tag{16}$$

Then

$$\delta = \tfrac{1}{2}\left[1 - \tfrac{1}{(N_r M)^{\alpha}}\right] \tag{17}$$

Then Eqs. (14) and (15) reduce to

$$1 = \tfrac{\alpha v}{4}\left(1-\delta\right)M\left[1+\delta\alpha\left(N_r M\right)^{\alpha}\right] \tag{18}$$

$$\delta = \tfrac{1}{2}\left[1-\tfrac{1}{\left(N_r M\right)^{\alpha}}\right] \tag{19}$$

Substituting (19) into (18), we obtain

$$1 = \tfrac{\alpha v}{4}\left(1+\left(N_r M\right)^{-\alpha}\right)M\left[1+\tfrac{1}{2}\left(1-\left(N_r M\right)^{-\alpha}\right)\alpha\left(N_r M\right)^{\alpha}\right] \tag{20}$$

Eq. (20) serves as the basis for our analysis of δ and σ.

First, about δ as claimed in (i). Denote

$$X := N_r M \tag{21}$$

and view the right-hand side of (20) as a function of X and N_r, $f(X, N_r)$, that is,

$$\begin{aligned} 1 &= \tfrac{\alpha v}{4N_t}\left(1+X^{-\alpha}\right)X\left[1+\tfrac{1}{2}\left(1-X^{-\alpha}\right)\alpha X^{\alpha}\right] \\ &= \tfrac{\alpha v}{4N_r}\left(X+X^{1-\alpha}\right)\left[\left(1+\tfrac{1}{2}\alpha\right)+\tfrac{1}{2}\alpha X^{\alpha}\right] \end{aligned} \tag{22}$$

Recall $0 < \alpha < 1$, which implies $1 - \alpha > 0$ and $1 - \frac{\alpha}{2} > 0$. Therefore, all the terms in the expression of $f(X; N_r)$ are both positive and monotonically nondecreasing. We have the following properties of $f(X; N_r)$:

(1) $f(0; N_r) = 0$; $f(\infty; N_r) = \infty$ for all v, $N_r > 0$.
(2) $f(X; N_r)$ increases strictly in X and decreases strictly in N_r.

Consequently, there exists a unique $X(N_r) > 0$ such that $f(X(N_r); N_r) = 1$. Clearly $X(N_r)$ monotonically increases in N_r due to the monotonicity of $f(X(N_r); N_r)$ w.r.t. X and N_r. By further expressing δ in terms of X via (19) and (21), $\delta = [1 - X^{-\alpha}]/2$, we see δ increases in X, thus in N_r. Moreover, the natural bound for interior $\delta > 0$ requires $X > 1$, which is equivalent to $f(1; N_r) < 1$ due to the monotonicity of f in X. By straightforward rearrangement, $f(1; N_r) < 1$ becomes Condition $R = (\alpha v)/2/N_r < 1$. This completes the proof of Part (i).

Now, consider σ. The uniqueness of $X > 1$ satisfying $f(X; N_r) = 1$ implies the uniqueness of σ. Indeed, by definitions of X, M, and S, we have $X = N_r M = N_r k S^{\alpha-1} = N_r k(\sigma V)^{\alpha-1}$. Under condition N_r, $K > 0$ and $X > 0$ according to the argument above, we have

$$\sigma = \left(\tfrac{N_r k}{X}\right)^{\frac{1}{\alpha-1}} \Big/ V > 0 \tag{23}$$

Therefore, it is never optimal for the platform to be completely closed, $\sigma = 0$ as long as v, N_r, $k > 0$. We now demonstrate the monotonicity property of σ with respect to N_r, or equivalently to δ, to complete the proof of Part (ii).

Noticing (23) can be rewritten as

$$\sigma = \left(\tfrac{k}{M}\right)^{\frac{1}{\alpha-2}} \Big/ V > 0 \tag{24}$$

We now convert $f(X; N_r)$ into a function of M and N_r. To be more precise, for

$$Q := \left(N_r\right)^{\alpha} \tag{25}$$

define the function

$$g(M;Q) := f(X;N_r) = \tfrac{\alpha v}{4}\left(M + M^{1-\alpha}/Q\right)\left[\left(1-\tfrac{1}{2}\alpha\right) + \tfrac{1}{2}\alpha Q M^{\alpha}\right] \tag{26}$$

Parallel to previous arguments, we have $g(0; Q) = 0$, $g(\infty; Q) = \infty$; thus, for all $Q > 0$, there exists a unique $M(Q) > 0$ such that $g(M(Q); Q) = 1$. The monotonicity property of $M(Q)$ w.r.t. Q is thus implied in the monotonicity of $g(M; Q)$ w.r.t. both M and Q.

As for the monotonicity of g, it is clear $g(M; Q)$ increases strictly in M. With respect to Q, consider the first-order partial derivative

$$\begin{aligned}\tfrac{\partial g}{\partial Q} &= \tfrac{\alpha v}{4}\left(-M^{1-\alpha}/Q^2\right)\left[\left(1-\tfrac{1}{2}\alpha\right)+\tfrac{1}{2}\alpha Q M^{\alpha}\right] + \tfrac{\alpha v}{4}\left(M + M^{-\alpha}/Q\right)\tfrac{1}{2}\alpha M^{a} \\ &= \tfrac{\alpha v}{4}\left(-M^{1-\alpha}/Q^2\right)\left(1-\tfrac{1}{2}\alpha\right) + \tfrac{\alpha v}{4}\tfrac{1}{2}\alpha M^{1+\alpha} \\ &= \tfrac{\alpha v}{4}\left[-\tfrac{\left(1-\frac{1}{2}\alpha\right)}{M^{\alpha}Q^2} + \tfrac{1}{2}\alpha M^{1+\alpha}\right]\end{aligned} \tag{27}$$

Clearly,

$$\begin{aligned}\left\{\tfrac{\partial g}{\partial Q} > 0\right\} &\Leftrightarrow \left(M^{\alpha}Q\right)^2 > \tfrac{(1-\alpha/2)}{\alpha/2} \\ &\Leftrightarrow M^{\alpha}Q > \sqrt{\tfrac{(1-\alpha)/2}{\alpha/2}} \\ &\Leftrightarrow (1-2\delta)^{-1} > \sqrt{\tfrac{(1-\alpha/2)}{\alpha/2}} \quad [by\,(19)] \\ &\Leftrightarrow \delta < \tfrac{1-\sqrt{\frac{\alpha}{2-\alpha}}}{2} = \bar{\delta}\end{aligned} \tag{28}$$

Combining equations $f(X; N_r) = 1$ and $\delta = [1 - X^{\alpha}]/2$, $\bar{\delta}$ uniquely determines an $\overline{N_d}$.

The monotonicity of δ w.r.t. N_r in Part (i) further yields

$$\left\{\tfrac{\partial g}{\partial Q} > 0\right\} \Leftrightarrow N_r < \overline{N_d} \tag{29}$$

Therefore, we conclude on $\left\{N_r < \overline{N_d}\right\}$ or $\left\{\delta < \bar{\delta}\right\}$,

$$\begin{aligned} g(M,Q) = 1 \Rightarrow M \downarrow Q \quad & [g \text{ increases in } M \text{ and in } Q] \\ \Leftrightarrow \sigma \uparrow Q \quad & [\text{by } (24)] \\ \Leftrightarrow \sigma \uparrow N_r \quad & [\text{by } (25)] \\ \Leftrightarrow \sigma \uparrow \delta \quad & [\text{monotonicity of } \delta \text{ w.r.t. } N_r \text{ in Part (i)}]\end{aligned} \tag{30}$$

In parallel, on $\left\{N_r \geq \overline{N_d}\right\}$ or $\left\{\delta \geq \bar{\delta}\right\}$, $\sigma \downarrow \delta, N_r$. Consequently, σ achieves its maximum at $\delta \geq \bar{\delta}$, $N_r = \overline{N_d}$. This completes the proof of Part (iii).

By combining Eqs. (19), (21), and (23) under condition $R < 1$, we can further express σ as a function of δ.

$$\sigma = \left(N_r k (1-2\delta)^{1/\alpha}\right)^{\frac{1}{1-\alpha}} \Big/ V \tag{31}$$

Clearly, $\sigma < 1$ is guaranteed by $(N_r k)^{1/(1-\alpha)}/V < 1$, or equivalently, $N_r k/V^{1-\alpha} = U < 1$. This confirms Part (ii) of the proposition. Finally, it is easy to see N_r monotonically increaess in N_d and $\omega = 1 - \rho$, and the proof is complete.