MIS Quarterly

RESEARCH ARTICLE

INTELLECTUAL PROPERTY RIGHTS AND CANNIBALIZATION IN INFORMATION TECHNOLOGY OUTSOURCING CONTRACTS

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Appendix

Derivation of Payoffs in Table 1

Let [(R_{ec}, R_{uc}), (R_{ev}, R_{uv})] represent an ownership structure where R_{ec} represents the client's excludability rights, R_{uc} represents the client's usability rights, R_{ev} represents the vendor's excludability rights, and R_{uv} represents the vendor's usability rights. Because of the diagonal symmetry only the bottom left diagonal will be discussed in detail.

[(E, U), (E, U)]

This ownership structure means that client and vendor each have the right to use the software as they see fit and to exclude all others. The client's usability rights gives him the full value of U_c, and the vendor's usability rights gives him the full value of U_v. Because each party must give its consent for a sale to occur, the parties proceed with Nash bargaining and gain one-half of the value of the sale.

[(E, U), (E, N)]

This ownership structure gives the client the right to both use and exclude, but only gives the vendor the right to exclude. This gives the client the full value of U_c, and one-half of the value from S and U_v. The vendor gains one-half of S and U_v. This occurs because no trade will occur without the agreement of both. If the client refuses to trade, then the client gains nothing. The same is true of the vendor. The client is already selling to everyone else in the market and, thus, he cannot credibly threaten to sell to someone else *instead* of the vendor. Rather, the client has to negotiate with the vendor in a situation where the vendor knows that the client will sell to everyone else regardless of whether he sells to the vendor. This means that the marginal impact of a sale to the vendor is the total amount of the sale. With physical property, which can be sold to only one entity, the marginal impact is the difference between the total amount of the sale and the amount a sale to the second best alterative would have garnered.

[(E, U), (N, U)]

This ownership structure gives the client the right to both use and exclude, but only gives the vendor the right to use. Thus, the client gains full value from U_c and S, and the vendor gains the full value of U_v.

[(E, U), (N, N)]

This ownership structure gives the client both excludability and usability rights, but grants no rights to the vendor. Not surprisingly, the client gains the full value of U_c and S. However, the vendor and client both gain one-half of U_v. This occurs because no trade will occur without the agreement of both. If the client refuses to trade, then the client gains nothing. The same is true of the vendor. Because the client is already selling to everyone else in the market, he has no credible threat to sell to someone else instead of the vendor.

[(E, N), (E, N)]

This ownership structure gives each party the right to exclude, but does not give the right of use to either. This means in the second period, when the contract is renegotiated each must grant the other the right to use, and each must agree to resale. This gives the client and vendor each one-half of U_c, one-half of U_v, and one-half of S.

[(E, N), (N, U)]

This ownership structure gives the client the right to exclude, but not use, and gives the vendor only the right to use. Clearly, this gives the vendor the full value of U_v. This also gives the client the full value of S. The client has the option to fail to exclude itself (and, as argued above, the client must exercise this right), but the vendor had no right to exclude the client and hence the client gains the full value of U_c.

[(E, N), (N, N)]

This ownership structure gives the client the right to exclude and no rights to the vendor. The client cannot exclude itself, but can exclude all others unilaterally, so the client gains the full value of U_c and S and one-half of U_v. The vendor gains one-half of U_v.

[(N, U), (N, U)]

This ownership structure gives each party the right to use but not to exclude. Thus, the client gains U_c, the vendor gains U_v, and neither party gains S. Within the bounds of this model, such an ownership structure is not Pareto rational because the parties could enrich themselves by including the excludability right.

[(N, U), (N, N)]

This ownership structure gives the client usability rights and no rights to the vendor. Thus, the client gains the full value of U_c and neither party gains either U_v or S. Again, this is not rational in this model.

[(N, N), (N, N)]

This last ownership structure gives no rights to either party, and thus neither party gains any benefit from such a contract. Clearly, this is not a rational contract.

First Order Conditions

Ownership Structure	Client Payoff	Client FOC	Vendor Payoff	Vendor FOC
[(E,U), (E,U)]	$U_c + ½S - p - i_c$	$\frac{\partial U_c}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = 1$	$U_v + ½S + p - i_v$	$\frac{\partial U_v}{\partial i_v} + \frac{1}{2}\frac{\partial S}{\partial i_v} = 1$
[(E,N), (E,U)]	$½U_c + ½S - p - i_c$	$\frac{1}{2}\frac{\partial U_c}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = 1$	$U_v + ½U_c + ½S + p - i_v$	$\frac{1}{2}\frac{\partial U_c}{\partial i_v} + \frac{\partial U_v}{\partial i_v} + \frac{1}{2}\frac{\partial S}{\partial i_v} = 1$
[(N,U), (E,U)]	$U_c - p - i_c$	$\frac{\partial U_c}{\partial i_c} = 1$	$U_v + S + p - i_v$	$\frac{\partial U_v}{\partial i_v} + \frac{\partial S}{\partial i_v} = 1$
[(N,N), (E,U)]	$½U_c - p - i_c$	$\frac{1}{2}\frac{\partial U_c}{\partial i_c} = 1$	$½U_c + U_v + S + p - i_v$	$\frac{1}{2}\frac{\partial U_c}{\partial i_v} + \frac{\partial U_v}{\partial i_v} + \frac{\partial S}{\partial i_v} = 1$
[(E,N), (E,N)]	$½U_c + ½U_v + ½S - p - i_c$	$\frac{1}{2}\frac{\partial U_c}{\partial i_c} + \frac{1}{2}\frac{\partial U_v}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = 1$	$½U_c + ½U_v + ½S + p - i_v$	$\frac{1}{2}\frac{\partial U_c}{\partial i_v} + \frac{1}{2}\frac{\partial U_v}{\partial i_v} + \frac{1}{2}\frac{\partial S}{\partial i_v} = 1$

Proof of Propositions

[(E,U), (E,U)]

> **Proposition 1**: If U_c is primarily sensitive to i_c, U_v is primarily sensitive to i_v, S depends equally on i_c and i_v, then [(E,U), (E,U)] ownership is the second best (i.e., it is at least as good as any other ownership structure, but fails to be optimal).

Under these assumptions, the first order conditions for the optimal investments become

$$\frac{\partial U_c(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_c(i_c)}{\partial i_c} + \frac{\partial S(i_c)}{\partial i_c} = 1 \tag{1}$$

and

$$\varepsilon \frac{\partial f_v(i_v)}{\partial i_v} + \frac{\partial U_v(i_v)}{\partial i_v} + \frac{\partial S(i_v)}{\partial i_v} = 1 \tag{2}$$

This makes use of the fact that $\partial U_j/\partial i_j = \partial f_j/\partial i_j$, where $j \in$ {client, vendor}. It can be seen that:

$$\lim_{\varepsilon \to 0}\left[\frac{\partial U_c(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_v(i_c)}{\partial i_c} + \frac{\partial S(i_c)}{\partial i_c}\right] = \frac{\partial U_c(i_c)}{\partial i_c} + \frac{\partial S(i_c)}{\partial i_c} \tag{3}$$

and

$$\lim_{\varepsilon \to 0}\left[\varepsilon \frac{\partial f_c(i_v)}{\partial i_v} + \frac{\partial U_v(i_v)}{\partial i_v} + \frac{\partial S(i_v)}{\partial i_v}\right] = \frac{\partial U_v(i_v)}{\partial i_v} + \frac{\partial S(i_v)}{\partial i_v} \tag{4}$$

Because (3) and (4) do not correspond to the first order conditions for [(E,U), (E,U)] in the table above, this ownership structure result is not optimal. Therefore, to show that it is second best, it is necessary to establish that it is superior to all other ownership structures.

First, compare the client's investment under [(E,N), (E,U)] to the client's investment under [(E,U), (E,U)].

$$\text{FOC [(E,U), (E,U)]:} \quad \frac{\partial U_c}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = g_c(i_c) = 1 \tag{5}$$

and

$$\text{FOC [(E,N), (E,U)]:} \quad \frac{1}{2}\frac{\partial U_c}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = 1 \Rightarrow g_c(i_c) = 1 + \frac{1}{2}\frac{\partial U_c}{\partial i_c} \tag{6}$$

Because U_c and S are twice continuously differentiable and concave, $g_c(i_c)$ is invertible and $\partial U_c/\partial i_c$ is positive. Concavity of U_c and S implies that $\partial g(i_c)/\partial i_c$ is negative. By the inverse function theorem $\partial i_c/\partial g_c(i_c) = [\partial g_c(i_c)/\partial i_c]^{-1}$, which is also negative. Therefore, increasing the value at which $g(i_c)$ is evaluated by the positive amount $\frac{1}{2}\partial U_c/\partial i_c$ decreases the level of investment that the client will make under [(E,N), (E,U)] as compared to [(E,U), (E,U)].

Next, compare the investments of the vendor under the two ownership structures. From the first order conditions for [(E,U), (E,U)], a function $g_v(i_v)$ can be constructed analogous to the $g_c(i_c)$ in (5). In this case the analogous equation for [(E,N), (E,U)] is

$$\text{FOC [(E,N), (E,U)]:} \quad g_v(i_v) = 1 + \frac{1}{2}\frac{\partial U_c}{\partial i_v} = 1 + \frac{1}{2}\varepsilon\frac{\partial f_c}{\partial i_v} \tag{7}$$

As ε approaches zero the later term vanishes and the first order conditions for [(E,N), (E,U)] and [(E,U), (E,U)] are the same and the level of investment each generates is the same.

This shows that the level of investment of the client is lower under [(E,N), (E,U)] than [(E,U), (E,U)] and the investment of the vendor is unchanged. However, it remains to be seen that this generates less value. To prove that it is sufficient to show that [(E,U), (E,U)] generates less investment than the first best. Then, because value is increasing in investment, [(E,U), (E,U)] will generate more value than [(E,N), (E,U)].

It is clear that functions $h_c(i_c)$ and $h_v(i_v)$ can be constructed from (3) and (4) analogous to $g_c(i_c)$ and $g_v(i_v)$ above, but describing the investment decision under the first best condition. One can then construct the first order conditions for [(E,U), (E,U)] as

$$\text{FOC [(E,U), (E,U)]:} \quad h_c(i_c) = 1 + \frac{1}{2}\frac{\partial S}{\partial i_c} \tag{8}$$

and

$$\text{FOC [(E,U), (E,U)]:} \quad h_v(i_v) = 1 + \frac{1}{2}\frac{\partial S}{\partial i_v} \tag{9}$$

Applying the logic above one more time shows that [(E,U), (E,U)] generates less investment than is optimal. Moreover, [(E,N), (E,U)] generates less investment than [(E,U), (E,U)] and hence is further from optimal. Therefore, [(E,U), (E,U)] is second best when compared to [(E,N), (E,U)].

Now it must be shown that the same holds for the other ownership choices. Next, compare [(E,U), (E,U)] to [(N,U), (E,U)]. It is clear that the client's investment is lower. This can be shown by repeating the same procedure as was used above. However, the vendor's investment is higher, again by the same logic. Therefore, it is necessary to show that increase in value created by the increase the vendor's investment is less than the absolute value of the decrease due to the client's decreased investment. To do this, first rewrite the first order conditions as

$$\text{FOC client:} \quad \frac{\partial U_c}{\partial i_c} + (1-x)\frac{\partial S}{\partial i_c} = 1 \tag{10}$$

and

$$\text{FOC vendor:} \quad \frac{\partial U_v}{\partial i_v} + x\frac{\partial S}{\partial i_v} = 1 \tag{11}$$

where $x = ½$ for [(E,U), (E,U)] and $x = 1$ for [(N,U), (E,U)].

Equations (10) and (11) represent implicit functions and can be analyzed using the implicit function theorem because the first term is always strictly positive. Applying the implicit function theorem yields

$$\frac{\partial i_c}{\partial x} = -\frac{-\dfrac{\partial S}{\partial i_c}}{\dfrac{\partial^2 U_c}{\partial i_c^2} + (1-x)\dfrac{\partial^2 S}{\partial i_c^2}} \tag{12}$$

and

$$\frac{\partial i_v}{\partial x} = -\frac{\dfrac{\partial S}{\partial i_v}}{\dfrac{\partial^2 U_v}{\partial i_v^2} + x\dfrac{\partial^2 S}{\partial i_v^2}} \tag{13}$$

The sign of $\partial i_v/\partial x$ is unambiguously positive because both of the bottom terms are negative and the top term is positive. The sign of $\partial i_v/\partial x$ is unambiguously negative for the same reasons.

By assumption $S(i_c, i_v) = f(i_c) + f(i_v)$, which implies that $\partial S/\partial i_c = \partial S/\partial i_v$, and $\partial^2 S/\partial i_c^2 = \partial^2 S/\partial i_v^2$. Thus, the numerators of $\partial i_c/\partial x$ and $\partial i_v/\partial x$ are equal. Because x ranges from ½ to 1, $\partial^2 U_c/\partial i_c^2 = (1 + \varepsilon)\partial^2 U_v/\partial i_v^2$, and both second partials are negative

$$\left|(1-x)\frac{\partial^2 S}{\partial i_c^2} - \frac{\partial^2 U_c}{\partial i_c^2}\right| \le \left|x\frac{\partial^2 S}{\partial i_v^2} - \frac{\partial^2 U_v}{\partial i_v^2}\right| \tag{14}$$

Because, the denominator of $\partial i_c/\partial x$ is smaller in absolute value than the denominator of $\partial i_v/\partial x$, $|\partial i_c/\partial x| > |\partial i_v/\partial x|$.

The effect of x on total profit is given by

$$\begin{aligned}\frac{\partial \pi}{\partial x} &= \frac{\partial U_c}{\partial i_c}\frac{\partial i_c}{\partial x} + \frac{\partial U_v}{\partial i_v}\frac{\partial i_v}{\partial x} + \frac{\partial S}{\partial i_c}\frac{\partial i_c}{\partial x} + \frac{\partial S}{\partial i_v}\frac{\partial i_v}{\partial x} - \frac{\partial i_c}{\partial x} - \frac{\partial i_v}{\partial x} \\ &= \frac{\partial i_c}{\partial x}\left(\frac{\partial U_c}{\partial i_c} + \frac{\partial S}{\partial i_c} - 1\right) + \frac{\partial i_v}{\partial x}\left(\frac{\partial U_v}{\partial i_v} + \frac{\partial S}{\partial i_v} - 1\right)\end{aligned} \tag{15}$$

By assumption $\partial U_c/\partial i_c = (1 + \varepsilon)\partial U_v/\partial i_v$, and as illustrated above $\partial S/\partial i_c = \partial S/\partial i_v$. By setting ε small enough the terms in brackets can be made arbitrarily close. Therefore, the sign of $\partial \pi/\partial x$ depends on the relative magnitude of $\partial i_c/\partial x$ and $\partial i_v/\partial x$. As shown above $\partial i_c/\partial x$ is positive and $\partial i_v/\partial x$ is negative and $\partial i_c/\partial x$ is higher in absolute value than $\partial i_v/\partial x$. Therefore $\partial \pi/\partial x < 0$.

The change in profit resulting in a change from [(E,U), (E,U)], where x = ½ to [(N,U), (E,U)], where x = 1 can be calculated via the fundamental theorem of calculus as

$$(\pi \mid x = 1) - (\pi \mid x = ½) = \int_{½}^{1} \frac{\partial \pi}{\partial x} dx < 0 \tag{16}$$

Therefore, the total profit under [(E,U), (E,U)] ownership, given the assumptions of this proposition, is greater than the profit under [(N,U), (E,U)]. Thus [(E,U), (E,U)] is second best relative to [(N,U), (E,U)].

To show that [(E,U), (E,U)] is second best relative to [(N,N), (E,U)] is trivial given the proof above. First, notice that given the assumptions of this proposition $\partial U_c/\partial i_v \to 0$. Thus, the vendor's incentive under [(N,N), (E,U)] is the same as the vendor's incentives under [(N,U), (E,U)]. However, the client's incentive is unambiguously smaller under [(N,N), (E,U)] than [(N,U), (E,U)]. Therefore, the joint profit under [(N,N), (E,U)] is less than the joint profit under [(N,U), (E,U)]. By transitivity, the joint profit under [(N,N), (E,U)] is less than the joint profit under [(E,U), (E,U)].

To compare [(E,U), (E,U)] to [(E,N), (E,N)] first note that $\partial U_c/\partial i_v$ and $\partial U_v/\partial i_c \to 0$. It is clear that functions analogous to $g_c(i_c)$ above can be constructed for both client and vendor. Applying the same logic as above, is it immediately obvious that the investment of both client and vendor will be lower and hence the joint profit will be lower.

Type [(E,U), (E,U)] ownership is superior to all other candidate ownership structures given the conditions of the proposition. Therefore, [(E,U), (E,U)] is second best, under the conditions of this proposition.

> **Proposition 2**: If U_v is primarily sensitive to i_v, U_c depends equally on i_c and i_v, S depends equally on i_c and i_v, the sensitivity U_c with respect to i_c is similar to the sensitivity U_c with respect to i_v, and U_v is sensitive over a wide range to i_v, then [(E,N), (E,U)] ownership is the second best.

The proof of this proposition follows the same steps as above. First write the first order conditions as

$$\text{FOC client:} \quad (1-y)\frac{\partial U_c}{\partial i_c} + (1-x)\frac{\partial S}{\partial i_c} = 1 \tag{17}$$

and

$$\text{FOC vendor:} \quad y\frac{\partial U_c}{\partial i_v} + \frac{\partial U_v}{\partial i_v} + x\frac{\partial S}{\partial i_v} = 1 \tag{18}$$

For type [(E,N), (E,U)] ownership $y = x = ½$. For type [(E,U), (E,U)] ownership $y = 1$ and $x = ½$. For the comparison between [(E,N), (E,U)] and [(E,U), (E,U)] x does not change and can be fixed at ½. Therefore, only the change with respect to y is important. Applying the implicit function theorem yields

$$\frac{\partial i_c}{\partial y} = -\frac{-\dfrac{\partial U_c}{\partial i_c}}{(1-y)\dfrac{\partial^2 U_c}{\partial i_c^2} + \dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_c^2}} \tag{19}$$

and

$$\frac{\partial i_v}{\partial y} = -\frac{\dfrac{\partial U_c}{\partial i_v}}{y\dfrac{\partial^2 U_c}{\partial i_v^2} + \dfrac{\partial^2 U_v}{\partial i_v^2} + \dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_v^2}} \tag{20}$$

By the assumptions of this section $\partial^2 U_c/\partial i_c^2$ is arbitrarily close to $\partial^2 U_c/\partial i_v^2$, $\partial U_c/\partial i_c$ is arbitrarily close to $\partial U_c/\partial i_v$ and $\partial^2 S/\partial i_c^2$ is equal to $\partial^2 S/\partial i_v^2$. Equations (19) and (20) evaluated at $y = ½$ can be written as

$$\frac{\partial i_c}{\partial y} = \frac{a}{-b} + \varepsilon' \tag{21}$$

and

$$\frac{\partial i_v}{\partial y} = \frac{-a}{-b + \dfrac{\partial^2 U_v}{\partial i_v^2}} \tag{22}$$

where a and b are positive and ε' is arbitrarily close to zero.

The effect of y on total profit is given by

$$\begin{aligned} \frac{\partial \pi}{\partial y} &= \frac{\partial U_c}{\partial i_c}\frac{\partial i_c}{\partial y} + \frac{\partial U_c}{\partial i_v}\frac{\partial i_v}{\partial y} + \frac{\partial U_v}{\partial i_v}\frac{\partial i_v}{\partial y} + \frac{\partial S}{\partial i_c}\frac{\partial i_c}{\partial y} + \frac{\partial S}{\partial i_v}\frac{\partial i_v}{\partial y} - \frac{\partial i_c}{\partial y} - \frac{\partial i_v}{\partial y} \\ &= \frac{\partial i_c}{\partial y}\left(\frac{\partial U_c}{\partial i_c} + \frac{\partial S}{\partial i_c} - 1\right) + \frac{\partial i_v}{\partial y}\left(\frac{\partial U_c}{\partial i_v} + \frac{\partial U_v}{\partial i_v} + \frac{\partial S}{\partial i_v} - 1\right) \end{aligned} \tag{23}$$

Again, by the assumptions of this section this can be rewritten as

$$\frac{\partial \pi}{\partial y} = \frac{a}{-b}(d) + \varepsilon + \frac{-a}{-b + \dfrac{\partial^2 U_v}{\partial i_v^2}}\left(d + \frac{\partial U_v}{\partial i_v}\right) \tag{Because}$$

Because ε' can be as close to zero as needed, $\partial \pi/\partial y > 0$ if

$$
\begin{aligned}
&\frac{a}{-b}(d) > \frac{a}{-b+\dfrac{\partial^2 U_v}{\partial i_v^2}}\left(d+\frac{\partial U_v}{\partial i_v}\right) \\
&\Rightarrow \frac{ad}{-b} > \frac{ad + a\dfrac{\partial U_v}{\partial i_v}}{-b+\dfrac{\partial^2 U_v}{\partial i_v^2}} \\
&\Rightarrow -bad + ad\frac{\partial^2 U_v}{\partial i_v^2} > -bad - ba\frac{\partial U_v}{\partial i_v} \\
&\Rightarrow d\frac{\partial^2 U_v}{\partial i_v^2} > -b\frac{\partial U_v}{\partial i_v} \\
&\Rightarrow \left| b\frac{\partial U_v}{\partial i_v} \right| > \left| d\frac{\partial^2 U_v}{\partial i_v^2} \right|
\end{aligned}
\tag{24}
$$

By assumption $\partial U^2_v/\partial i_v^2 = \varepsilon \partial U_v/\partial i_v$, so that the term on the right can be made arbitrarily small relative to the term on the left.

Thus, $\partial \pi/\partial x > 0$ for $0 = y = ½$. The change in profit by switching y from 0 to ½ can be calculated as

$$
(\pi \mid x = ½) - (\pi \mid x = 0) = \int_0^{½} \frac{\partial \pi}{\partial x}\,dx > 0 \tag{25}
$$

Therefore, type [(E,N), (E,U)] ownership is better than type [(E,U), (E,U)] under the assumptions of this section.

Next, compare [(E,N), (E,U)] to [(N,U), (E,U)]. In this case, it is simple to use (10) thru (16) to compare [(N,U), (E,U)] to [(E,U), (E,U)] instead, and then use transitivity to show that [(E,N), (E,U)] > [(E,U), (E,U)] > [(N,U), (E,U)]. It is necessary to note that the assumptions $S(i_c, i_v) = f(i_c) + f(i_v)$, $\partial^2 U_c/\partial i_c^2 = (1 + \varepsilon)\partial^2 U_v/\partial i_v^2$, and $\partial U_c/\partial i_c = (1 + \varepsilon)\partial U_v/\partial i_v$ hold under this section. Therefore the results in (10) thru (16) hold in this section and by transitivity [(E,N), (E,U)] is better than [(N,U), (E,U)] in the sense that joint surplus is greater.

To compare [(E,N), (E,U)] to [(N,N), (E,U)], it is easiest to again use transitivity and show that [(N,N), (E,U)] offers the same incentive to the vendor as [(N,U), (E,U)]. This occurs because $\partial U_c/\partial i_v = \varepsilon f(i_v)$, and thus, in the limit the incentives are the same. However, [(N,N), (E,U)] offers the strictly less incentive to the client than [(N,U), (E,U)]. Hence, the logic of (5) and (6) can be applied to show that [(N,U), (E,U)] generates more joint surplus than [(N,N), (E,U)]. Transitivity then shows that [(E,N), (E,U)] generates more joint surplus than [(N,N), (E,U)].

Finally, the logic of (5) and (6) can be applied to show that [(E,N), (E,U)] generates more joint surplus than [(E,N), (E,N)]. It must first be noted that, by assumption, $\partial U_v/\partial i_c = \varepsilon f(i_c)$. This implies that, in the limit, the client's first order conditions are not changed. However, the vendor is offered strictly less incentive to invest. Therefore, [(E,N), (E,U)] generates more joint surplus than [(E,N), (E,N)].

Under the assumptions of this section type [(E,N), (E,U)] ownership is superior to all other alternative ownership structures. Hence, under these assumptions type [(E,N), (E,U)] ownership is second best.

> **Proposition 3**: If U_c is primarily sensitive to i_c, and both U_v and S are primarily sensitive to i_v then [(E,N), (E,U)] ownership is the second best.

This is easily shown by noting that the first order conditions for first best under these assumptions are

$$\text{FOC client:} \quad \frac{\partial U_c(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_v(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_s(i_c)}{\partial i_c} = 1 \tag{26}$$

and

$$\text{FOC vendor:} \quad \varepsilon \frac{\partial f_c(i_v)}{\partial i_v} + \frac{\partial U_v(i_v)}{\partial i_v} + \frac{\partial S(i_v)}{\partial i_v} = 1 \tag{27}$$

As ε approaches zero these first order conditions converge to the first order conditions given for type [(N,U), (E,U)] ownership. Thus, by choosing ε arbitrarily small, the investment levels for type [(N,U), (E,U)] ownership can be made arbitrarily close to the investment levels for first best.

None of the other first order conditions offers the same levels of investment. Therefore, by choosing ε arbitrarily small, the investment levels for type [(N,U), (E,U)] ownership can be made arbitrarily closer to first best than any other ownership structure under the assumptions of this section.

> **Proposition 4**: If all sources of value depend primarily on the vendor's i_v, then [(N,N), (E,U)] ownership is the second best.

To see this first note that under these assumptions

$$\text{FOC client:} \quad \begin{aligned} &\varepsilon \frac{\partial f_c(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_v(i_c)}{\partial i_c} + \varepsilon \frac{\partial f_s(i_c)}{\partial i_c} = 1 \\ &\Rightarrow \frac{\partial f_c(i_c)}{\partial i_c} + \frac{\partial f_v(i_c)}{\partial i_c} + \frac{\partial f_s(i_c)}{\partial i_c} = \frac{1}{\varepsilon} \end{aligned} \tag{28}$$

Clearly, by choosing ε arbitrarily small the investment level of the client can be made arbitrarily small. See the logic following (6) for a more detailed explanation. Thus, all of the ownership structures offer the client arbitrarily small incentive for investment.

Following the logic of (5) and (6), all other ownership structures offer the vendor strictly less incentive to invest. Therefore, the vendor invests more and the client invests no less under type [(N,N), (E,U)] ownership than any other ownership structure, but less than the optimal level of investment. Hence, type [(N,N), (E,U)] ownership generates more joint surplus than any other ownership structure, making it second best.

> **Proposition 5**: If the value from all sources depends equally on i_c and i_v, the return to i_c with respect to U_c falls off more rapidly than the return to i_v with respect to U_c, the return to i_v with respect to U_v falls off more rapidly than the return to i_c with respect to U_v, then [(E,N), (E,N)] ownership is the second best.

To prove this it is necessary to compare to each other the other ownership structures individually. First, compare to [(E,U), (E,U)] by writing the first order conditions as

$$\text{FOC client:}\quad x\frac{\partial U_c}{\partial i_c}+(1-x)\frac{\partial U_v}{\partial i_c}+\frac{1}{2}\frac{\partial S}{\partial i_c}=1 \tag{29}$$

and

$$\text{FOC vendor:}\quad (1-x)\frac{\partial U_c}{\partial i_v}+x\frac{\partial U_v}{\partial i_v}+\frac{1}{2}\frac{\partial S}{\partial i_v}=1 \tag{30}$$

For [(E,U), (E,U)] x = 1 and for [(E,N), (E,N)] x = ½. Following the logic developed above.

$$\frac{\partial i_c}{\partial x}=-\frac{\dfrac{\partial U_c}{\partial i_c}-\dfrac{\partial U_v}{\partial i_c}}{x\dfrac{\partial^2 U_c}{\partial i_c^2}+(1-x)\dfrac{\partial^2 U_v}{\partial i_c^2}+\dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_c^2}} \tag{31}$$

and

$$\frac{\partial i_v}{\partial x}=-\frac{\dfrac{\partial U_v}{\partial i_v}-\dfrac{\partial U_c}{\partial i_v}}{(1-x)\dfrac{\partial^2 U_c}{\partial i_v^2}+x\dfrac{\partial^2 U_v}{\partial i_v^2}+\dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_v^2}} \tag{32}$$

Given that $\partial U_j/\partial i_k > \partial U_j/\partial i_j$ both partials are unambiguously negative. The effect of x on joint profit can be written as

$$\begin{aligned}\frac{\partial \pi}{\partial x}&=\frac{\partial U_c}{\partial i_c}\frac{\partial i_c}{\partial x}+\frac{\partial U_c}{\partial i_v}\frac{\partial i_v}{\partial x}+\frac{\partial U_v}{\partial i_v}\frac{\partial i_v}{\partial x}+\frac{\partial U_v}{\partial i_c}\frac{\partial i_c}{\partial x}+\frac{\partial S}{\partial i_c}\frac{\partial i_c}{\partial x}+\frac{\partial S}{\partial i_v}\frac{\partial i_v}{\partial x}-\frac{\partial i_c}{\partial x}-\frac{\partial i_v}{\partial x}\\&=\frac{\partial i_c}{\partial x}\left(\frac{\partial U_c}{\partial i_c}+\frac{\partial U_v}{\partial i_c}+\frac{\partial S}{\partial i_c}-1\right)+\frac{\partial i_v}{\partial x}\left(\frac{\partial U_v}{\partial i_v}+\frac{\partial U_c}{\partial i_v}+\frac{\partial S}{\partial i_v}-1\right)\end{aligned} \tag{33}$$

Substituting the first order conditions into the first terms in parentheses and recognizing that (1-x) is non-negative yields

$$\begin{aligned}&\left(\frac{\partial U_c}{\partial i_c}+\frac{\partial U_v}{\partial i_c}+\frac{\partial S}{\partial i_c}-\left(x\frac{\partial U_c}{\partial i_c}+(1-x)\frac{\partial U_v}{\partial i_c}+\frac{1}{2}\frac{\partial S}{\partial i_c}\right)\right)\\&=(1-x)\frac{\partial U_c}{\partial i_c}+x\frac{\partial U_v}{\partial i_c}+\frac{1}{2}\frac{\partial S}{\partial i_c}>0\end{aligned} \tag{34}$$

A similar substitution shows that the second term in parentheses in (33) is strictly positive. Thus, $\partial\pi/\partial x$ is strictly negative. This means that the integral is strictly negative, which means that the profit at $x = 1$ is less than the profit at $x = ½$. Therefore, [(E,N), (E,N)] is better than [(E,U), (E,U)].

Next, compare to [(E,N), (E,U)], by writing the first order conditions as

$$\text{FOC client: } \frac{1}{2}\frac{\partial U_c}{\partial i_c} + (1-x)\frac{\partial U_v}{\partial i_c} + \frac{1}{2}\frac{\partial S}{\partial i_c} = 1 \qquad (35)$$

and

$$\text{FOC vendor: } \frac{1}{2}\frac{\partial U_c}{\partial i_v} + x\frac{\partial U_v}{\partial i_v} + \frac{1}{2}\frac{\partial S}{\partial i_v} = 1 \qquad (36)$$

This suggests that

$$\frac{\partial i_c}{\partial x} = -\frac{-\dfrac{\partial U_v}{\partial i_c}}{\dfrac{1}{2}\dfrac{\partial^2 U_c}{\partial i_c^2} + (1-x)\dfrac{\partial^2 U_v}{\partial i_c^2} + \dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_c^2}} \qquad (37)$$

and

$$\frac{\partial i_v}{\partial x} = -\frac{\dfrac{\partial U_v}{\partial i_v}}{\dfrac{1}{2}\dfrac{\partial^2 U_c}{\partial i_v^2} + x\dfrac{\partial^2 U_v}{\partial i_v^2} + \dfrac{1}{2}\dfrac{\partial^2 S}{\partial i_v^2}} \qquad (38)$$

At $x = ½$, $\partial i_c/\partial x < 0$ and $\partial i_v/\partial x > 0$. Given that $\partial U_j/\partial i_k > \partial U_j/\partial i_j$, $|\partial i_c/\partial x| > |\partial i_v/\partial x|$. As x increases from ½ to 1 the denominator of (38) increases making $\partial i_v/\partial x$ smaller in absolute value. At the same time the denominator of (37) decreases making $\partial i_c/\partial x$ larger in absolute value. Thus, for all x between ½ and 1, inclusive, $|\partial i_c/\partial x| > |\partial i_v/\partial x|$.

The effect of x on joint profit can be written as (33). It is clear from (33) that $\partial\pi/\partial x$ is strictly negative. By the prior logic [(E,N), (E,N)] is better than [(E,N), (E,U)].

To compare with [(N,U), (E,U)] write the first order conditions as

$$\text{FOC client: } x\frac{\partial U_c}{\partial i_c} + (1-x)\frac{\partial U_v}{\partial i_c} + (1-x)\frac{\partial S}{\partial i_c} = 1 \qquad (39)$$

and

FOC vendor: $(1-x)\dfrac{\partial U_c}{\partial i_v} + x\dfrac{\partial U_v}{\partial i_v} + x\dfrac{\partial S}{\partial i_v} = 1$ (40)

This suggests that

$$\frac{\partial i_c}{\partial x} = -\frac{\dfrac{\partial U_c}{\partial i_c} - \dfrac{\partial U_v}{\partial i_c} - \dfrac{\partial S}{\partial i_c}}{x\dfrac{\partial^2 U_c}{\partial i_c^2} + (1-x)\dfrac{\partial^2 U_v}{\partial i_c^2} + x\dfrac{\partial^2 S}{\partial i_c^2}} \tag{41}$$

and

$$\frac{\partial i_v}{\partial x} = -\frac{-\dfrac{\partial U_c}{\partial i_v} + \dfrac{\partial U_v}{\partial i_v} + \dfrac{\partial S}{\partial i_v}}{(1-x)\dfrac{\partial^2 U_c}{\partial i_v^2} + x\dfrac{\partial^2 U_v}{\partial i_v^2} + (1-x)\dfrac{\partial^2 S}{\partial i_v^2}} \tag{42}$$

$\partial i_c/\partial x$ is unambiguously negative. $\partial i_v/\partial x$ is negative if $\partial U_c/\partial i_v > \partial U_v/\partial i_v + \partial S/\partial i_v$. Because $\partial U_c/\partial i_v > \partial U_v/\partial i_v + q$, this is true for $q > \partial S/\partial i_v$. Thus, by setting q large enough $\partial i_v/\partial x$ is unambiguously negative. Following the logic above as x changes from ½ to 1, the joint profit decreases. Thus, [(E,N), (E,N)] is better than [(N,U), (E,U)].

Finally, compare [(E,N), (E,N)] to [(N,N), (E,U)] by writing the first order conditions as

FOC client: $\dfrac{1}{2}\dfrac{\partial U_c}{\partial i_c} + (1-x)\dfrac{\partial U_v}{\partial i_c} + (1-x)\dfrac{\partial S}{\partial i_c} = 1$ (43)

and

FOC vendor: $\dfrac{1}{2}\dfrac{\partial U_c}{\partial i_v} + x\dfrac{\partial U_v}{\partial i_v} + x\dfrac{\partial S}{\partial i_v} = 1$ (44)

This suggests that

$$\frac{\partial i_c}{\partial x} = -\frac{-\dfrac{\partial U_v}{\partial i_c} - \dfrac{\partial S}{\partial i_c}}{\dfrac{1}{2}\dfrac{\partial^2 U_c}{\partial i_c^2} + (1-x)\dfrac{\partial^2 U_v}{\partial i_c^2} + (1-x)\dfrac{\partial^2 S}{\partial i_c^2}} \tag{45}$$

and

$$\frac{\partial i_v}{\partial x} = -\frac{\dfrac{\partial U_v}{\partial i_v} + \dfrac{\partial S}{\partial i_v}}{\dfrac{1}{2}\dfrac{\partial^2 U_c}{\partial i_v^2} + x\dfrac{\partial^2 U_v}{\partial i_v^2} + x\dfrac{\partial^2 S}{\partial i_v^2}} \tag{46}$$

At $x = ½$, $\partial i_c/\partial x < 0$ and $\partial i_v/\partial x > 0$. Given that $\partial U_j/\partial i_k > \partial U_j/\partial i_j$, $|\partial i_c/\partial x| > |\partial i_v/\partial x|$. As x increases from ½ to 1 the denominator of (45) increases making $\partial i_v/\partial x$ smaller in absolute value. At the same time the denominator of (46) decreases making $\partial i_c/\partial x$ larger in absolute value. Thus, for all x between ½ and 1, inclusive, $|\partial i_c/\partial x| > |\partial i_v/\partial x|$.

The effect of x on joint profit can be written as (33). It is clear from (33) that $\partial \pi/\partial x$ is strictly negative. By the prior logic [(E,N), (E,N)] is better than [(E,N), (E,U)].

> **Proposition 6**: If the net benefit to client use of the software is $[U_c(i_c, i_v) - \delta S(i_c, i_v)]$, then the client will have reduced incentive to invest under each of the five ownership structures.

The client's investment decision solves

$$\text{FOC client:} \quad \frac{\partial \tilde{U}_c}{\partial i_c} + \frac{f(i_c)}{\partial i_c} = 1 \tag{47}$$

where $\tilde{U} = U_c$ in the absence of cannibalization and $U_c - \delta S$ in the presence of cannibalization, and f = some combination of U_c and S. From this it can be seen that in the absence of cannibalization the client's investment solves:

$$\text{FOC client:} \quad \frac{\partial U_c}{\partial i_c} + \frac{f(i_c)}{\partial i_c} = 1 \tag{48}$$

and with cannibalization the client's investment solves

$$\text{FOC client:} \quad \begin{aligned} &\frac{\partial U_c}{\partial i_c} - \delta\frac{\partial S}{\partial i_c} + \frac{f(i_c)}{\partial i_c} = 1 \\ &\Rightarrow \frac{\partial U_c}{\partial i_c} + \frac{f(i_c)}{\partial i_c} = 1 + \delta\frac{\partial S}{\partial i_c} \end{aligned} \tag{49}$$

Because δ is positive, by the logic above the solution to (48) is greater than the solution to (49). Hence, the client invests less in the presence of cannibalization.

> **Proposition 7**: If the net benefit to client use of the software is $[U_c(i_c, i_v) - \delta S(i_c, i_v)]$ then possibility of vendor *over*investement occurs under each of the five ownership structures.

In the presence of cannibalization the vendor's *first best* investment solves

$$\left[\frac{\partial U_c(i_v)}{\partial i_v} - \delta \frac{\partial S(i_v)}{\partial i_v}\right] + \frac{\partial U_v(i_v)}{\partial i_v} + \frac{\partial S(i_v)}{\partial i_v} = g(i_v) = 1 \tag{50}$$

The coefficients *x*, *y*, and *z* are between zero and one depending on the ownership structure. However, in the presence of cannibalization, the vendor's actual investment solves

$$\begin{aligned} & x\left[\frac{\partial U_c(i_v)}{\partial i_v} - \delta \frac{\partial S(i_v)}{\partial i_v}\right] + y\frac{\partial U_v(i_v)}{\partial i_v} + z\frac{\partial S(i_v)}{\partial i_v} = 1 \\ & = g(i_v) - (1-x)\frac{\partial U_c(i_v)}{\partial i_v} - (1-y)\frac{\partial U_v(i_v)}{\partial i_v} - (1 - z + zx\delta - \delta)\frac{\partial S(i_v)}{\partial i_v} \\ & \Rightarrow g(i_v) = 1 + \left[(1-x)\frac{\partial U_c(i_v)}{\partial i_v} + (1-y)\frac{\partial U_v(i_v)}{\partial i_v} + (1 - z + zx\delta - \delta)\frac{\partial S(i_v)}{\partial i_v}\right] \end{aligned} \tag{51}$$

By the logic above the level of i_v that solves (51) is greater than the level of i_v that solves (50) if the term in brackets is negative. This can only be true if $zx < 1$. The ownership structures under consideration allow $0 = z = 1$ and $0 = x = ½$, which implies $0 = zx = ½$. Thus, the term in brackets is negative if

$$\begin{aligned} & \left[(1-x)\frac{\partial U_c(i_v)}{\partial i_v} + (1-y)\frac{\partial U_v(i_v)}{\partial i_v} + (1 - z + zx\delta - \delta)\frac{\partial S(i_v)}{\partial i_v}\right] < 0 \\ & \Rightarrow (1-x)\frac{\partial U_c(i_v)}{\partial i_v} + (1-y)\frac{\partial U_v(i_v)}{\partial i_v} + (1-z)\frac{\partial S(i_v)}{\partial i_v} < (\delta - zx\delta)\frac{\partial S(i_v)}{\partial i_v} \\ & \Rightarrow \delta > \frac{(1-x)\frac{\partial U_c(i_v)}{\partial i_v} + (1-y)\frac{\partial U_v(i_v)}{\partial i_v} + (1-z)\frac{\partial S(i_v)}{\partial i_v}}{(1-zx)\frac{\partial S(i_v)}{\partial i_v}} \end{aligned} \tag{52}$$

By assumption, all the terms to the right of the inequality are positive, and the only restriction on δ is $\delta > 0$. This implies that there exists some δ for which the solution to (51) is greater the solution to (50). Thus, there is a possibility of vendor overinvestment in the presence of cannibalization.

> **Proposition 8**: If the cannibalization problem is defined to be greater when the level of δ necessary for vendor overinvestment is smaller and $\partial U_c/\partial_v$ is arbitrary large, then the ownership structures can be ranked from the largest cannibalization problem to the smallest as [(E,U), (E,U)], [(N,U), (E,U)], [(E,N), (E,N)], [(E,N), (E,U)], [(N,N), (E,U)].

To show this the minimum δ needed to satisfy (52) need to be compared for each ownership structure. These solutions are detailed below.

Ownership Structure	δ
[(E,U), (E,U)]	$\delta^1 > \dfrac{(1)\dfrac{\partial U_c(i_v)}{\partial i_v} + (0)\dfrac{\partial U_v(i_v)}{\partial i_v} + (½)\dfrac{\partial S(i_v)}{\partial i_v}}{(½)\dfrac{\partial S(i_v)}{\partial i_v}}$
[(E,N), (E,U)]	$\delta^2 > \dfrac{(½)\dfrac{\partial U_c(i_v)}{\partial i_v} + (0)\dfrac{\partial U_v(i_v)}{\partial i_v} + (½)\dfrac{\partial S(i_v)}{\partial i_v}}{(¾)\dfrac{\partial S(i_v)}{\partial i_v}}$
[(N,U), (E,U)]	$\delta^3 > \dfrac{(1)\dfrac{\partial U_c(i_v)}{\partial i_v} + (0)\dfrac{\partial U_v(i_v)}{\partial i_v} + (0)\dfrac{\partial S(i_v)}{\partial i_v}}{(½)\dfrac{\partial S(i_v)}{\partial i_v}}$
[(N,N), (E,U)]	$\delta^4 > \dfrac{(½)\dfrac{\partial U_c(i_v)}{\partial i_v} + (0)\dfrac{\partial U_v(i_v)}{\partial i_v} + (0)\dfrac{\partial S(i_v)}{\partial i_v}}{(1)\dfrac{\partial S(i_v)}{\partial i_v}}$
[(E,N), (E,N)]	$\delta^5 > \dfrac{(½)\dfrac{\partial U_c(i_v)}{\partial i_v} + (½)\dfrac{\partial U_v(i_v)}{\partial i_v} + (½)\dfrac{\partial S(i_v)}{\partial i_v}}{(¾)\dfrac{\partial S(i_v)}{\partial i_v}}$

It is clear that δ^4 is the smallest because the denominator is the largest and the numerator is the smallest. It is also clear that $\delta^5 > \delta^2$ because they have the same denominator and δ^5 has a larger numerator. Likewise, $\delta^1 > \delta^3$ because they have the same denominator and δ^5 has a larger numerator. Given that $\partial U_c/\partial i_v$ is large $\delta^3 > \delta^5$ because the numerator of δ^3 can be made large faster than the numerator of δ^5. That is, the limit of δ^3/δ^5 as $\partial U_c/\partial i_v$ approaches infinity is 3/2. Thus, $\delta^1 > \delta^3 > \delta^5 > \delta^2 > \delta^4$, which means that the cannibalization problem, as defined by the minimum δ is largest for [(E,U), (E,U)], second largest for [(N,U), (E,U)], third largest for [(E,N), (E,N)], fourth largest for [(E,N), (E,U)], and smallest for [(N,N), (E,U)].